Bank Information Sharing and Liquidity Risk*

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Abstract

We propose a novel rationale for the existence of bank information sharing schemes. Banks may voluntarily disclose borrowers' credit history to maintain asset market liquidity. By sharing such information, banks mitigate adverse selection when selling their loans in secondary markets. This reduces the cost of asset liquidation in case of liquidity shocks. Information sharing arises endogenously when the liquidity benefit dominates the cost of losing market power in the primary loan market competition. We show banks having incentives to truthfully disclose borrowers' credit history, even if such information is non-verifiable. We also provide a rationale for promoting public credit registries.

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1 Introduction

One of the reasons for the existence of banks is that they provide the service of liquidity transformation by borrowing short-term and lending long-term. The funding liquidity risk is a natural by-product of the banks' *raison d'être* (Diamond and Dybvig, 1983). This paper argues that such funding liquidity risk can also be a reason why banks share credit information of their borrowers. The need for information sharing arises because banks in need of liquidity may have to sell their assets in secondary markets, and information asymmetry in such markets can make asset liquidation costly. Information sharing allows banks to reduce adverse selection in secondary loan markets, which in turn reduces the damage of premature liquidation.[1]

The benefit of information sharing, however, has to be traded off with its potential cost. Letting other banks learn the credit worthiness of its own borrowers, an incumbent bank can sacrifice its market power as an information monopoly. As its competitors forcefully compete for good borrowers, the incumbent bank would extract less rent from its borrowers in primary loan markets.[2] Our paper provides a throughout analysis of this trade-off.

Our theory of bank information sharing is motivated by observations of US consumer credit markets (such as markets for mortgages and credit cards). These markets are competitive and contestable. At the same time, banks are able to sell loans originated in these markets. We believe that the two features can be linked and both related to credit information sharing. On the one hand, the shared information on a borrower's credit history—typically summarized by a FICO score—reduces the asymmetric information about the borrower's creditworthiness. This enables banks to compete for the borrower with whom they have no previous lending relationship. On the other hand, the resulting loan is more marketable because the information contained in the FICO score signals the borrower's creditworthiness so that potential buyers in secondary loan markets are less concerned about the adverse selection. In sum, the shared credit information both intensifies primary loan market competition and promotes secondary market liquidity.

[1] While our formal model focuses on an asset sale, similar arguments can be made for collateralized borrowing or securitization, where the reduced adverse selection will lead to lower haircuts or lower private credit enhancement. We discuss such interpretations in Section 5.6.

[2] The concern for primary loan market competition is empirically documented in Liberti, Sturgess and Sutherland (2018) for the credit market of equipment finance in the U.S..

The observation that credit information sharing has helped to promote securitization in the U.S. has inspired European regulators. In their effort to revive the securitization market in the post-crisis Europe, the European Central Bank and the Bank of England have jointly pointed out that "credit registers could also improve the availability and quality of information that could, in principle, also benefit securitization markets by allowing investors to build more accurate models of default and recovery rates" (BoE and ECB, 2014).[3] Our paper provides theoretical supports that credit information sharing schemes can indeed promote asset marketability by reducing information asymmetry, and such schemes are sustainable as it can be in banks' own interests to truthfully share the information.

The bank in our model has two basic features. First, it has private information about its borrowers, in the form of their types (intrinsic credit worthiness) and credit history (repayment records). Second, the bank faces funding liquidity risk of potential runs. Upon runs of its creditors, the bank will need to liquidate its loans to meet the liquidity need. When the loan quality is unknown to outsiders, the price of the loan in the secondary market would be lower than its fundamental value due to adverse selection. This may cause a bank with high quality loan to fail, which destroys value both from a private and a social point of view. Credit information sharing would provide a way to mitigate adverse selection and avoid costly liquidation.

Our main analysis focuses on bank sharing borrowers' credit history and unfolds in three steps.[4] First, we show that sharing verifiable credit history can boost the price of the loan in the secondary market. This is a non-trivial result because information sharing has two countervailing effects. On the one hand, the shared credit history mitigates the adverse selection problem, which tends to boost the loan price. On the other hand, the shared credit history intensifies competition in the primary loan market. This lowers the face value of the loan, which tends to reduce its price. We show that the former effect always dominates. Moreover, we establish conditions under which credit information sharing is efficient. Under such conditions, the shared credit history sufficiently boosts the loan price

[3]Note that in the proposal, the credit registers would provide information both to primary-market lenders as well as to secondary-market investors. The same is true for credit information such as the FICO score in the U.S. markets. Our model also incorporates this feature.

[4]To illustrate the main trade-off between secondary-market liquidity and rent-extraction in the primary market, we start our analysis with a simplest setting where the bank can share verifiable information about the borrower's type and show that credit information sharing can arise endogenously.

in the secondary market so that the bank can survive runs. Whereas without information sharing, the loan price is low because of adverse selection, and the bank fails in runs.

Second, we show that the bank voluntarily commits to sharing the borrower's verifiable credit history when the benefit of higher asset liquidity exceeds the cost of losing market power. Naturally, the conditions for the bank to find credit information sharing privately profitable are (at least weakly) stricter than the conditions under which information sharing is efficient.

Third, we relax the assumption that the borrower's credit history, once shared, is verifiable.[5] With unverifiable credit history, the bank may overstate the borrower's past credit performances to obtain a higher price when the loan is on sale. Nevertheless, we show the bank still has incentives to truthfully reveal its borrower's previous default, when the truthful disclosure allows the bank to extract more rent from the borrower. Naturally, guaranteeing truth-telling imposes an extra (ex-post) incentive constraint and tightens the conditions under which information sharing endogenously emerges.[6]

Since the bank does not always find it profitable to share its borrowers' credit history even when it is efficient, our model provides a rationale to promote public credit registries. In particular, when the information is verifiable, requiring banks to share borrower's credit history improves efficiency. More interestingly, even if credit history is unverifiable, a public registry can still improve efficiency. While a bank can find it privately unprofitable to share its borrowers' credit history, the bank will disclose such information truthfully once it is obliged to do so.

We contribute to the literature of bank credit information sharing in three ways. First,

[5] Such an assumption can be restrictive from a theoretical point of view. Also, the recent study of Giannetti et al. (2017) shows that the assumption does not always hold empirically. The authors show that banks manipulated their internal credit ratings of their borrowers before reporting to Argentinian credit registry. On a more casual level, information manipulation can also take place in the form of 'zombie' lending, like it occurred in Japan with the ever-greening phenomenon or in Spain where banks kept on lending to real estate firms likely to be in distress after housing market crash. Notice such countries are characterized by relatively low competition in the credit market. As we will show later, this is coherent with our model which predicts banks operating in competitive markets have stronger incentives to truthfully reveal their borrowers' credit information.

[6] An alternative and established way to sustain truth telling is to consider a dynamic game where the incumbent bank has some reputation at stake. We show that truth telling can be sustained in equilibrium even in a static game.

we show that a bank's decision to share its borrower's credit information can be driven by the concern of funding liquidity. Our conjecture focuses on the liability side of the bank's balance and differs from the theories that emphasize credit information sharing's role in reducing credit risk on the asset side of the bank's balance sheet. Second, we relax the common assumption of verifiable credit history in the literature and show that truthful information sharing can still be sustained. Finally, we highlight the potential efficiency gains in establishing public credit registries.

The theory literature of credit information sharing is dedicated to understand why banks voluntarily share their borrowers' credit information while they can profit from such proprietary information. Our conjecture that information sharing is driven by market liquidity is novel and complementary to existing theories. Previous literature has mostly explained the existence of information sharing by focusing on the primary loan market. In their seminal paper, Pagano and Jappelli (1993) rationalize information sharing as a mechanism to reduce adverse selection in primary loan markets. Exchanging credit information about borrowers reduces the riskiness of banks' assets and increases banks' expected profits. Similarly, information sharing can mitigate borrowers' moral hazard problems (Padilla and Pagano, 1997 and 2000). We see information sharing not only stemming from frictions in the primary loan market but also from frictions in the secondary market. The two explanations are not mutually exclusive.

Another strand of the literature argues that information sharing allows the incumbent bank to extract more monopolistic rent. When competition for borrowers occurs in two periods, inviting competitors to enter in the second period by sharing information can dampen the competition in the first period (Bouckaert and Degryse, 2004; Gehrig and Stenbacka 2007). Sharing information about a borrower's past defaults also deters the entry of competitors, which allows the incumbent bank to capture the borrower (Bouckaert and Degryse, 2004). This mechanism is also present in our model, and it is instrumental in sustaining truth-telling when the borrower's credit history is not verifiable.

Other than providing expositions for voluntary credit information sharing, the literature has also examined how information sharing affects banks' lending strategies. For example, information sharing can complement collateralization since banks are able to impose high collateral requirement after high-risk borrowers are identified via information sharing (Karapetyan and Stacescu 2014b). Information sharing can also induce information acquisition. Indeed, collecting soft information becomes an urgent task for the bank to keep its profits after hard information has been communicated (Karapetyan and Stacescu 2014a).

The empirical studies on information sharing, following the existing theoretical literature, have mostly focused on the impact of credit registries on banks' credit risk exposures and firms' access to bank financing. For example, Djankov, McLiesh and Shleifer (2007) shows how private credit increases after the introduction of credit registries, in particular their positive role is found in developing countries. Brown et al. (2009) show that information sharing improves credit availability and lower cost of credit to firms in transition countries. Houston et al. (2010) find that information sharing is correlated with lower bank insolvency risk and likelihood of financial crisis. Doblas-Madrid and Minetti (2013) provide evidence that information sharing reduces contract delinquencies.

It is important to notice that credit information sharing schemes (i.e., private credit bureaus and public credit registries) differ from other forms of credit information disclosure such as credit ratings. While credit information sharing mostly involves the reporting of borrowers' *historical* records of repayments, credit ratings are rating agencies' subjective predictions about borrowers' risk of *future* defaults. Furthermore, rating agencies also use historical credit information (e.g., FICO scores) as a relevant input for their models. More importantly, credit ratings can be an equilibrium outcome of complicated strategic interactions between the rating agencies and the rated institution (e.g., Bolton, Freixas and Shapiro 2012). Instead, in our model the shared credit history is a noisy signal about a borrower's creditworthiness derived from the borrower's record of repayments. Accordingly, the decision to participate in a credit information sharing scheme can be viewed from the perspective of the Bayesian persuasion literature pioneered by Kamenica and Gentzkow (2011). The incumbent bank's engagement in information sharing can be interpreted as using its borrower's credit history as a noisy signal to persuade asset buyers to bid a sufficiently high price for the bank's loan so as to survive runs.[7]

The remainder of this paper is organized as follows. Section 2 presents the model. Section 3 determines the conditions under which information sharing arises endogenously. We first assume that the information shared is on the borrower's type (Section 3.1), then on the borrower's verifiable credit history (Section 3.2) and, finally, on the borrower's non-

[7] Another application of Bayesian persuasion in finance is analyzed by Goldstein and Leitner (2018). The authors study how a similar engagement can explain banks' commitment to disclose the results of stress test.

verifiable credit history (Section 3.3). Section 4 provides a rationale for the establishment of public credit registries. Section 5 discusses several robustness and extensions. Section 6 concludes. The Appendix collects the proofs.

2 The Model

We consider a four-period economy with dates $t = 0, 1, 2, 3, 4$. The economy is populated by the following agents: two banks (an incumbent bank and an entrant bank), a borrower, and many depositors as well as potential buyers of bank assets. All agents are risk neutral. The gross return of the risk-free asset is equal to r_f.

The borrower needs a loan of unit size at $t = 2$. The loan pays off at $t = 4$, and its return depends on the type of the borrower. The borrower can be either safe (H-type) or risky (L-type). The common prior on the borrower's type is that $\Pr(H) = \alpha$ and $\Pr(L) = 1 - \alpha$. A safe borrower generates a payoff $R > r_f$ with certainty,[8] whereas a risky borrower generates a payoff that depends on an aggregate state $s \in \{G, B\}$, which realizes at $t = 3$ and is publicly observable. In the good state G, a risky borrower generates the same payoff R as a safe borrower, but the borrower only generates a payoff of 0 in the bad state B. The probabilities of the two states are $\Pr(G) = \pi$ and $\Pr(B) = 1 - \pi$, respectively. One can interpret the H-type being a prime mortgage borrower and the L-type being a subprime borrower. While both can pay back their loans in a housing boom ($s = G$), the subprime borrowers will default in a sluggish housing market ($s = B$).

The incumbent bank has an established lending relationship with the borrower and privately observes both the borrower's type (i.e., the creditworthiness) and credit history (i.e., the repayment record) at $t = 1$. We denote a credit history with a default record by D and a credit history with no previous default by $\overline{D}$. While the safe borrower has a credit history $\overline{D}$ with probability 1, the risky borrower has a credit history $\overline{D}$ with probability δ and a credit history D with probability $1 - \delta$. One may interpret the default as a late repayment on the borrower's debt (for example, credit card). While the safe type never misses a repayment, the risky type incurs a late repayment with probability $1 - \delta$.[9]

[8] We show in Section 5.3 that our results can be generalized to allow the H-type to be risky.

[9] Notice that the realization of the aggregate state s is independent of the credit history of the borrower which captures an idiosyncratic risk. In the example of mortgage loans, the probability of a housing market boom is independent of the borrower's repayment record on, for example, his credit card debt.

We model credit information sharing as a unilateral decision of the incumbent bank at $t = 0$. We denote the information sharing regime by $i \in \{N, S\}$, where N refers to the regime without information sharing and S as the regime with information sharing. When $i = S$, the incumbent bank makes a public announcement at $t = 1$ about the borrower's credit information. We take a progressive approach when it comes to what kind of information the incumbent bank can share. We start by analyzing a simplest scenario where the information on the borrower's type can be shared and is verifiable. This allows us to illustrate the main mechanism of our model. For the main part of our analysis, we consider a setting where only the borrower's credit history can be shared and is verifiable.[10] As a final step, we relax the assumption on the verifiability of the credit history and allow for the possibility that the incumbent bank can overstate the borrower's past loan performance.[11]

The entrant bank has no lending relationship with the borrower. It observes no information about the borrower's type or credit history unless the incumbent bank decides to share such information. At $t = 2$, the entrant bank can compete for the borrower by offering competitive loan rates, but to initiate the new lending relationship it has to pay a fixed cost c. Such a cost instead represents a sunk cost for the incumbent bank.[12]

The bank that wins the loan market competition will be financed solely by deposits. Depositors are assumed to be price-takers who only demand to earn the risk-free rate r_f in expectation. The winner bank holds the market power to set the deposit rate, but we assume perfect market discipline so that deposit rates reflect the bank's riskiness. This allows us to abstract from the risk-shifting incentives induced by equity holders' limited liability.[13] Similar to the entrant bank, depositors have no private information about the

[10] In the banking literature, the borrower's type can be considered as soft information which is difficult to communicate to third parties. Instead, credit history can be considered as hard information that can be shared with outsiders.

[11] We assume that the incumbent bank cannot falsely claim a borrower to have a default record when the borrower has none. This is because the borrower would have the incentives and means to correct it. For example, borrowers can access their own credit record and correct such inaccuracy under Fair Credit Act in the US. A false report may result in a legal dispute.

[12] The fixed cost c can be interpreted as the cost that the entrant bank has to bear to establish new branches, to hire and train new staffs, or to comply to any financial regulations. Alternatively, it can represent the borrower's switching cost that is born by the entrant bank.

[13] We discuss the implications of relaxing this assumption in Section 5.2.

borrower and learn the borrower's type or credit history if the incumbent bank shares such information. In case the incumbent bank wins the loan market competition, the depositors may also infer the borrower's type from the deposit rate offered by the incumbent bank.[14]

To capture the funding liquidity risk, we assume that the incumbent bank faces a run at $t = 3$ with a probability ρ.[15] We assume the risk of run to be idiosyncratic and independent of the aggregate state s. When the run happens, all depositors withdraw their funds, and the incumbent bank has to raise liquidity to meet the depositors' withdrawals.[16]

Upon the run, the bank can sell the loan in a competitive secondary market. We assume that the loan is indivisible and the bank has to sell it as a whole.[17] Buyers in the secondary loan market are risk-neutral and demand to break even in expectation. They observe the state s that realizes in $t = 3$, but do not have private information on the borrower. Nor do the buyers observe the loan rate or the deposit rate. Therefore, while they condition their bids on the state, they can condition their bids on the borrower's type or credit history only if the incumbent bank shared such information in $t = 1$.

It is the incumbent bank's private information whether it faces a run or not. The loan can be on sale for two reasons: either due to funding liquidity needs, in which case an H-type loan can be on sale, or due to a strategic sale for arbitrage, in which case only an L-type loan will be on sale. The possibility of a strategic asset sale leads to adverse selection in the secondary asset market. An H-type loan will be underpriced in an asset sale, so that the incumbent bank that holds an H-type loan can fail due to illiquidity. In the case of a bank failure, we assume that bankruptcy costs result in a zero salvage value. Since the shared information reduces adverse selection and may boost secondary-market loan price,

[14] In other words, the incumbent may signal its loan quality to the depositors via the deposit rates. This signaling game is analyzed in the proof of Lemma 1 and 2. Throughout the paper we assume that the depositors have an off-equilibrium belief worse than the prior. This includes the standard assumption that when an off-equilibrium action is observed, the players have the worst belief about the loan quality.

[15] While the incumbent bank faces the liquidity risk the entrant bank does not face such risk. Notice however that we give the incumbent bank the possibility to manage liquidity risk by (unilaterally) share information on the borrower. The model setup is symmetric in this respect. We discuss in Section 5.4 the case in which information sharing may emerge as mutual agreement among banks.

[16] One can think that bank run is triggered by a sun-spot event as in Diamond and Dybvig (1983). This is also a feature of bank runs based on global games with arbitrarily precise private signal. When a run happens, then all depositors run on the bank. We discuss the robustness of our mechanism to endogenous bank run risks in Section 5.7.

[17] We discuss the robustness of our mechanism to alternative assumptions on loan sale in Section 5.6.

the incumbent bank can have the incentive to disclose borrower's credit information.

We make the following two parametric assumptions:

$$c + r_f < R; \tag{1}$$

$$c + r_f > \frac{r_f}{\pi}. \tag{2}$$

Assumption (1) states that the entrant bank finds it profitable to lend to an H-type borrower. Assumption (2) determines that the cost of entry is relatively high, and it is satisfied for low value of c when π is high.[18] Assumptions (1) and (2) jointly imply

$$R > \frac{r_f}{\pi}. \tag{3}$$

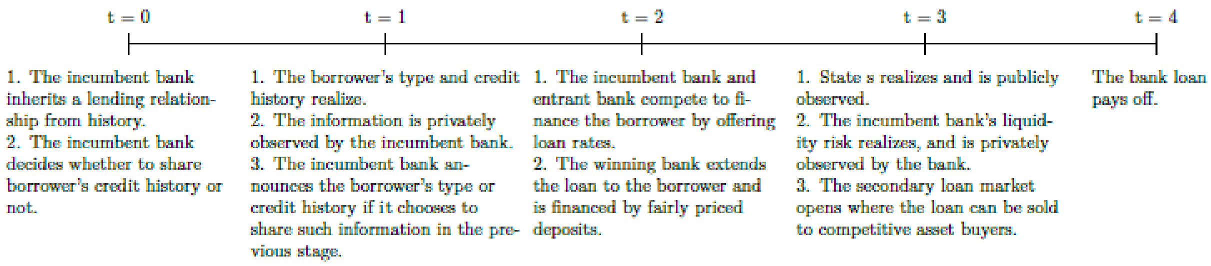
That is, both the L- and H-type loans have positive NPVs, so that the incumbent bank finds it profitable to lend to the borrower independent of the borrower's type. Condition (3) is necessary for the analysis of credit information sharing. Suppose that it does not hold and that the incumbent bank only finds it profitable to finance the H-type borrower. Then the fact that the incumbent bank decides to lend fully reveals that the borrower is an H-type, so that credit information sharing would not play any role.

The sequence of events is summarized in Figure 1. The timing captures the fact that information sharing is a long-term decision (commitment), while the competition in the

[18] As it will be clear later, assumption (2) guarantees a unique equilibrium of our model. In particular, it avoids the existence of an equilibrium where the entrant bank finances the loan if the borrower is an H-type, while the incumbent bank finances the loan if the borrower is an L-type. In the example of mortgage loans, assumption (2) is satisfied for low cost c when the probability of a housing market boom is sufficiently large.

loan market and the liquidity risk faced by the bank are shorter-term concerns.[19]

Figure 1: Time line of the model



3 Equilibrium Information Sharing

Notice that, once the incumbent bank has chosen an information sharing regime $i \in \{N, S\}$, we face a well-defined game g_i that we can solve backwards. Therefore, we can determine the incumbent bank's payoffs in each self-contained game g_i. The incumbent bank would choose at $t = 0$ the information sharing regime i that delivers the highest expected payoff.

In Section 3.1, the incumbent bank is assumed to be able to share verifiable information on the borrowers' *type*. The assumption allows us to illustrate the main mechanism of our model in a most simple manner. We employ in Section 3.2 a more realistic assumption that the incumbent bank can share the *credit history* of the borrower, under the hypothesis that such information is verifiable. We show that in both scenarios the incumbent bank has incentives to share credit information.[20] Finally, in Section 3.3, we also allow the credit history to be *unverifiable*. We show that truthful disclosure can be sustained.

3.1 Sharing Borrower Type

To illustrate the main mechanism of our model, we start by considering the simplest setting,

[19] Note that it is necessary to assume that information sharing decision ($t = 0$) is made *before* the incumbent bank acquires the borrower's information ($t = 1$). Otherwise, information sharing decision itself may serve as a signaling device of the incumbent bank.

[20] We show in Section 5.1 that the incumbent bank prefers to share the borrower's credit history even if it can credibly share also the borrower's type.

where the incumbent bank can share verifiable information on the borrower's type.

Let us first examine the case where the incumbent bank operates under the regime $i = S$ and the shared information reveals the borrower being the H-type. The game $g_S(H)$ features complete information, and we solve for its subgame perfect equilibrium (SPE) by backward induction. First, we determine the secondary-market price for an $H-$type loan in aggregate state G and B, respectively. Second, we compute the deposit rate at which depositors are willing to supply their funds to the banks, given that the shared information identifies the borrower as the H-type. Finally, we determine the loan rate at which the bank offers credit to the borrower.

Since the safe type never defaults, asset buyers' competitive bidding drives the price of the H-type loan up to its face value in the secondary loan market. Let $P_S^B(H)$ and $P_S^G(H)$ be the price of the H-type loan under regime $i = S$ in state B and G, respectively. We have

$$P_S^B(H) = P_S^G(H) = R_S^*(H),$$

where $R_S^*(H)$ denotes the equilibrium loan rate for the H-type borrower under regime $i = S$.

Depositors understand that a loan given to an H-type borrower does not default. Thus, they perceive lending to a bank that finances such a loan safe, and are willing to accept the risk-free rate. Let $r_S^I(H)$ and $r_S^E(H)$ denote the deposit rates that the incumbent bank and the entrant bank, respectively, need to offer. We have

$$r_S^I(H) = r_S^E(H) = r_f.$$

The entrant bank also understands that an H-type borrower is risk-free and can break even by offering a loan rate $R_S^E(H)$ that equals its cost of deposits $r_S^E(H)$ plus its entry cost c. The equilibrium loan rate $R_S^*(H)$ is determined by the value of the loan return R. If $R \geq c + r_f = R_S^E(H)$, the incumbent bank has to match the entrant bank's loan rate, resulting in an equilibrium loan rate $c + r_f$. Whereas if $R < c + r_f$, the entrant bank does not find profitable to bid for the borrower and the equilibrium loan rate will hit a corner solution of R. Therefore, the equilibrium loan rate for an H-type borrower is as follows:[21]

$$R_S^*(H) = \min\{R, c + r_f\}.$$

The incumbent bank's profit from revealing the borrower as the H-type is denoted by[22]

$$\Pi_S(H) = R_S^*(H) - r_S^I(H).$$

Consider now the shared information reveals the borrower being an L-type. The game $g_S(L)$ still features complete information and its SPE can be solved by backward induction. Since the L-type succeeds in state G but defaults in state B, the loan price on the secondary market will be state-dependent. It equals zero when $s = B$ and the face value of the loan when $s = G$. That is,

$$P_S^B(L) = 0 \quad \text{and} \quad P_S^G(L) = R_S^*(L),$$

where $R_S^*(L)$ denotes the equilibrium loan rate for the L-type borrower.

Depositors understand that the price of an L-type loan is zero when $s = B$, so that they will be repaid only in the favorable state G which occurs with a probability π. The depositors consider their lending to a bank that finances an L-type loan risky, and would accept the following deposit rates

$$r_S^I(L) = r_S^E(L) = \frac{r_f}{\pi} > r_f.$$

Expecting to recoup its investment only when $s = G$, the entrant bank can break even by offering a loan rate

$$R_S^E(L) = \frac{c + r_f}{\pi},$$

so that its expected payoff equals the cost of lending, $\pi \cdot \left[R_S^E(L) - r_S^E(L)\right] + (1 - \pi) \cdot 0 = c$. Again, depending on whether $R_S^E(L)$ is higher or lower than R, the equilibrium loan rate

[21] Recall that both the deposit rate and the loan rate are set before the aggregate state s realizes. Therefore, they cannot be conditional on s.

[22] In the paper, we use Π to denote the incumbent bank' expected profit at $t = 2$, that is, the expected payoff *after* the bank receives the information about the borrower's type and credit history. The uncertainty is due to the aggregate state and potential liquidity risk. We denote the bank's expected profit at $t = 0$ by V, that is, the expected payoff *before* the bank receives any private information on the borrower.

for the L-type borrower can be written as

$$R_S^*(L) = \min\left\{R, \frac{c + r_f}{\pi}\right\}.$$

Provided that the borrower is an L-type, the incumbent bank's expected profit is

$$\Pi_S(L) = \pi \cdot \left[R_S^*(L) - r_S^I(L)\right].$$

When the information sharing decision is made, the type of the borrower is unknown. The incumbent bank's expected profit at $t = 0$ can be written as follows:[23]

$$V_S^{Type} = \alpha\Pi_S(H) + (1-\alpha)\Pi_S(L) = \alpha R_S^*(H) + (1-\alpha)\pi R_S^*(L) - r_f. \tag{4}$$

We now turn to the case where the bank operates under the regime $i = N$, that is, the bank does not share any information. The game g_N features now incomplete information. All outsiders (the entrant bank, depositors and asset buyers) need to form beliefs about the quality of the borrower, and we solve the game backwards using the concept of perfect Bayesian equilibrium (PBE).[24]

The secondary loan market now features adverse selection. With no information on borrower's type and a belief that both types of borrowers are financed, asset buyers has a prior that the loan on sale is an H-type with a probability α. In state B, the incumbent bank will sell an H-type loan only if it is hit by the liquidity shock, but will always sell an L-type loan to arbitrage with its private information. Taking into account the strategic asset sale of the incumbent bank, the asset buyers' break-even price in state B when purchasing a loan with unknown quality can be written as

$$P_N^B = \frac{\alpha\rho}{(1-\alpha) + \alpha\rho} R_N^*,$$

where R_N^* denotes the equilibrium loan rate under the regime of no information sharing. In state G, the asset buyers understand that both H- and L-type borrowers can repay the loan, so that

$$P_N^G = R_N^*.$$

[23] When calculating the bank's expected profit at $t = 0$, we suppress the incumbent bank's payoff from the existing lending, i.e., the borrower's repayment at $t = 1$. Since the repayment is from pre-existing lending and does not change across information sharing regimes, suppressing it does not affect our analysis of bank's information sharing decision.

[24] We refer to Appendix .1 for the formal definition of the equilibrium notion adopted in game g_N.

Suppose $P_N^B < r_f$ (i.e., the price of the loan on sale in state B is insufficient to cover the risk-free rate). Then, depositors at the incumbent bank understand that their bank is risky in state B: The bank can repay its deposit funding, only if it holds an H-type loan and experiences no run. Whereas the incumbent bank can always repay its deposit funding in state G. Therefore, to finance itself, the incumbent bank needs to offer

$$r_N^I = \frac{r_f}{\pi + (1-\pi)\alpha(1-\rho)}.$$

On the other hand, since the entrant bank is assumed to face no liquidity risk, the depositors are willing to accept a deposit rate

$$r_N^E = \frac{r_f}{\pi + (1-\pi)\alpha}.$$

Given the funding cost r_N^E and a belief that both H- and L-type borrowers participate in the market, the entrant bank will break even by offering a loan rate

$$R_N^E = \frac{c + r_f}{\alpha + (1-\alpha)\pi},$$

such that $\alpha\left(R_N^E - r_N^E\right) + (1-\alpha)\pi\left(R_N^E - r_N^E\right) = c$. Again, depending on whether R_N^E of the entrant bank is higher or lower than R, the equilibrium loan rate R_N^* can be written as

$$R_N^* = \min\left\{R, \frac{c + r_f}{\alpha + (1-\alpha)\pi}\right\}.$$

The following Lemma establishes that when $P_N^B < r_f$, the game g_N has a unique PBE.

Lemma 1 *When $P_N^B < r_f$, there exists a unique PBE of the game g_N. The incumbent bank offers to the borrower a pooling loan rate*

$$R_N^* = \min\left\{R, R_N^E\right\},$$

regardless of the type or credit history. Here

$$R_N^E = \frac{c + r_f}{\alpha + (1-\alpha)\pi}$$

is the entrant bank's break-even rate when lending to a borrower of unknown type and credit history. The incumbent bank offers a risky deposit rate

$$r_N^I = \frac{r_f}{\pi + (1-\pi)\alpha(1-\rho)}$$

that allows depositors to break even. The incumbent bank sells its H-type loan only if hit by the liquidity shock in state B. Upon the loan sale, buyers offer state-contingent prices

$$P_N^G = R_N^* \quad \textit{and} \quad P_N^B = \frac{\alpha\rho}{(1-\alpha)+\alpha\rho} R_N^*.$$

Proof. See Appendix A.1. ■

Lemma 1 shows that there exists a set of parameters in which the unique pure-strategy PBE involves the incumbent bank lending to the borrower regardless of his type. This implies that, on the equilibrium path, the secondary market for the loan features adverse selection and the incumbent bank fails when it operates under the regime of no information sharing in state B—even when holding a loan granted to the H-type borrower.

Under the regime of no information sharing, the incumbent bank's expected profit at $t = 0$ can be written as

$$V_N = \pi\left(R_N^* - r_N^I\right) + (1-\pi)\alpha(1-\rho)\left(R_N^* - r_N^I\right) = [\pi + (1-\pi)\alpha(1-\rho)] \cdot R_N^* - r_f. \quad (5)$$

Comparing expressions (4) and (5) leads to the following proposition.

Proposition 1 *There exists a critical $\hat{\rho}$, such that when $\rho > \hat{\rho}$ and $P_N^B < r_f$, the incumbent bank prefers to share the information on borrowers' type to no information sharing.*

Proof. See Appendix A.2. ■

To gain the intuition, notice that the proposition is easily verified when all equilibrium loan rates take either the interior or the corner solutions. Indeed, in these two cases, the proposition holds independent of the level of ρ. Specifically, consider the all-corner-solution case: $R_S^*(L) = R_S^*(H) = R_N^* = R$. We have

$$V_S^{Type} = [\alpha + (1-\alpha)\pi]R - r_f > V_N = [\pi + (1-\pi)\alpha(1-\rho)]R - r_f.$$

The difference in payoffs, $V_S^{Type} - V_N = (1-\pi)\alpha\rho \cdot R > 0$, captures that, by sharing the borrower's type, the bank holding an H-type loan avoids the failure when facing a run in state B. Intuitively, sharing information does not negatively affect the market power of the incumbent bank when all the loan rates are equal to R. The incumbent bank only benefits from the reduced adverse selection, which makes sharing the borrower's type preferable. On the other hand, when the equilibrium loan rates are all interior solutions (i.e., equal to the break-even rate of the entrant bank), we have $V_S^{Type} = c > V_N$. Intuitively, this is a scenario in which the primary market is always contestable, independent of whether the entrant bank knows the borrower's type. As a result, the incumbent bank always makes a profit c, which equals the entrant bank's entry cost. Again sharing borrower's type only brings the benefit of reduced adverse selection. In sum, in these two cases, sharing borrower's type dominates no information sharing, independent of ρ. For intermediate cases, the result still holds when the risk of run is sufficiently high.

This simple case where the incumbent bank can directly share the borrower's type illustrates the main mechanism of our paper. Information sharing reduces adverse selection and boosts the price of H-type loans. In state B, the price increases from $P_N^B < r_f$ to $P_S^B(H) > r_f$, rescuing the incumbent bank with an H-type loan from runs. On the other hand, the incumbent bank can lose market power, as the loan rate charged to the borrower drops from R_N^* to $R_S^*(H)$. Intuitively, the benefit of information sharing is more prominent when the liquidity risk is sufficiently high, so that the benefit exceeds its cost.

3.2 Sharing Verifiable Credit History

We now analyze the more realistic scenario in which borrower's type cannot be shared but only borrower's credit history can be communicated to third parties. Clearly, when the incumbent bank does not share information on the borrower credit history we have again the game g_N, whose PBE is characterized in Lemma 1.

Under the information sharing regime, we first consider the case in which the incumbent bank discloses a credit history of no default (i.e., a $\overline{D}$-history). This announcement only partially reveals the borrower's type, as the borrower can still be either an H- or L-type.[25] The game $g_S(\overline{D})$ therefore features incomplete information. Parallel to Lemma 1, the following Lemma characterizes the unique pure strategy PBE of the incomplete information game $g_S(\overline{D})$.

Lemma 2 *When $P_S^B(\overline{D}) > r_f$, there exists a unique PBE of the game $g_S(\overline{D})$. The incumbent bank offers to the borrower who has no previous default a loan rate*

$$R_S^*(\overline{D}) = \min\left\{R, R_S^E(\overline{D})\right\},$$

[25] Buyers in this market price the assets depending on hard information (credit history) even though bad loans sometime are sold. For example, Keys *et al.* (2010) find that delinquency rates are higher for loans with FICO scores just above 620 as compared to loans with FICO scores just below 620.

regardless of the borrower's type. Here

$$R_S^E(\overline{D}) = \frac{\alpha + (1-\alpha)\delta}{\alpha + (1-\alpha)\delta\pi}(c + r_f)$$

is the entrant bank's break-even rate of lending to a borrower with no previous default. The incumbent bank offers a risk-free deposit rate $r_S^I(\overline{D}) = r_f$ and sells the H-type loan only if hit by the liquidity shock in state B. Upon the loan sale, asset buyers offer state-contingent prices

$$P_S^G(\overline{D}) = R_S^*(\overline{D}) \quad \textit{and} \quad P_S^B(\overline{D}) = \frac{\alpha\rho}{(1-\alpha)\delta + \alpha\rho} R_S^*(\overline{D}).$$

Proof. See Appendix A.3. ■

Lemma 2 shows that there exists a set of parameters in which the unique pure-strategy PBE involves the incumbent bank financing the loan at $t = 2$ regardless of the borrower's type. Moreover, on the equilibrium path, the bank can survive a run even in state B.

When the incumbent bank announces that the borrower has a previous default (i.e., a D-history), the shared information reveals the borrower as an L-type. The game $g_s(D)$ therefore features complete information and can be solved with SPE, similarly to the case where the incumbent bank announces the borrower being an L-type as analyzed in Section 3.1. The following Lemma characterizes the unique SPE of the game $g_S(D)$.

Lemma 3 *When the borrower has a credit history of previous default, there exists a unique SPE of the game $g_S(D)$. The incumbent bank offers to the borrower a loan rate*

$$R_S^*(D) = \min\left\{R, R_S^E(D)\right\}.$$

Here $R_S^E(D) = (c + r_f)/\pi$ is the entrant bank's break-even rate when lending to a borrower who has previously defaulted. The incumbent bank offers a risky deposit rate $r_S^I(D) = r_f/\pi$ that allows depositors to break even. Upon the loan sale, asset buyers offer state-contingent prices

$$P_S^G(D) = R_S^*(D) \quad \textit{and} \quad P_S^B(D) = 0.$$

Proof. See Appendix A.4. ■

Lemma 3 characterizes the unique SPE of the game $g_S(D)$, where the incumbent bank lends to the borrower of a D-history at $t = 2$, and fails at $t = 3$ when the state B occurs.

Comparing the equilibrium loan rates in Lemma 1, 2 and 3, we can rank them as follows:

$$R_S^*(\overline{D}) \leq R_N^* \leq R_S^*(D). \tag{6}$$

The equalities hold only when all the equilibrium loan rates hit the corner solution R (i.e., the entrant bank does not find profitable to bid for the borrower). Intuitively, the borrower with a default (D-history) is identified as an L-type and charged the highest loan rate accordingly. On the other hand, the borrower with no previous default ($\overline{D}$-history) is more likely to be an H-type. Correspondingly, the loan rate drops. When no information is shared, the equilibrium loan rate will be set according to the prior probabilities of the borrower types, resulting in the intermediate loan rate R_N^*.

Depending on how the project income R relates to the ranking of the equilibrium loan rates in (6), we have the following four cases $j = 0, 1, 2, 3$:

- Case 0: $R \in \mathbb{R}_0 \equiv \left[c + r_f, R_S^E(\overline{D})\right)$ then $R_S^*(\overline{D}) = R_N^* = R_S^*(D) = R$.
- Case 1: $R \in \mathbb{R}_1 \equiv \left[R_S^E(\overline{D}), R_N^E\right)$ then $R_S^*(\overline{D}) = R_S^E(\overline{D})$ and $R_N^* = R_S^*(D) = R$.
- Case 2: $R \in \mathbb{R}_2 \equiv \left[R_N^E, R_S^E(D)\right)$ then $R_S^*(\overline{D}) = R_S^E(\overline{D})$, $R_N^* = R_N^E$ and $R_S^*(D) = R$.
- Case 3: $R \in \mathbb{R}_3 \equiv \left[R_S^E(D), +\infty\right)$ then $R_S^*(\overline{D}) = R_S^E(\overline{D})$, $R_N^* = R_N^E$ and $R_S^*(D) = R_S^E(D)$.

Notice that we index each case with the number of interior solutions (i.e., when the equilibrium loan rates is determined by the bid of the entrant bank). Each case shows a different degree of loan market contestability. In Case 0, the project payoff R is so low that the entrant bank finds it unprofitable to enter the market even if the borrower has no previous default. In Case 3, on the other hand, R is so high that the entrant bank competes even for the borrower who previously defaulted. The higher R, the more contestable the primary loan market. The four mutually exclusive cases are illustrated in Figure 2.

[Insert Figure 2 here]

The benefit of sharing credit history.

We now show that, in each of the four cases, there exists a set of parameters where the incumbent bank holding a loan with a $\overline{D}$-history survives a run in state B when sharing information and fails otherwise. That is, information sharing is beneficial since it saves the incumbent bank from illiquidity.

As we pointed out in Section 3.1, information sharing can boost the secondary-market loan price in state B. Indeed, without information sharing, asset buyers bid according to the prior belief about the loan quality and take into account of adverse selection. The incumbent bank can therefore fail in a run–even if it holds the H-type loan. When the incumbent bank shares the credit history, the perceived loan quality is higher for a loan with no previous default $\overline{D}$, which mitigates adverse selection and boosts the secondary-market loan price.

Sharing credit history also has a countervailing impact on the secondary-market loan price. Once it is known that the borrower has no previous default, the incumbent bank may charge a loan rate lower than the one under no information sharing, because the entrant bank now competes for the borrower more fiercely. As the loan on sale has a lower face value, information sharing *may* result in $P_S^B(\overline{D}) < P_N^B$. The following Lemma shows, however, that the mitigation of adverse selection dominates the drop in the face value of the loan, so that sharing credit history *always* increases the price of a loan with $\overline{D}$-history in state B.

Lemma 4 *The equilibrium prices of the loan on sale are such that $P_S^B(\overline{D}) > P_N^B$. That is, in state B, the price in the secondary market for a loan with a $\overline{D}$-history is higher than the price for a loan with an unknown credit history. There exists a range of parameters such that $P_S^B(\overline{D}) > r_f > P_N^B$. That is, in state B, the incumbent bank holding a loan of a $\overline{D}$-history can survive a run, while the bank fails without information sharing.*

Proof. See Appendix A.5. ■

To provide the intuition, we discuss here Case 2, a core case upon which the complete proof builds.[26] Recall that in Case 2 the equilibrium loan rates are $R_N^* = R_N^E$ and $R_S^*(\overline{D}) = R_S^E(\overline{D})$. Substituting them into the expressions that characterize the equilibrium prices P_N^B and $P_S^B(\overline{D})$, as given in Lemma 1 and 2 respectively, we have

$$\frac{P_N^B}{P_S^B(\overline{D})} = \underbrace{\frac{(1-\alpha)\delta + \alpha\rho}{(1-\alpha) + \alpha\rho}}_{(A)} \cdot \underbrace{\frac{\alpha + (1-\alpha)\delta\pi}{(\alpha + (1-\alpha)\pi)\,(\alpha + (1-\alpha)\delta)}}_{(B)}.$$

This ratio between P_N^B and $P_S^B(\overline{D})$ can be decomposed into the product of two elements. Expression (A) reflects how information sharing affects the information asymmetry in the secondary loan market, and expression (B) captures the impact of information sharing on

[26] In Case 0, the equilibrium loan rates are $R_S^*(\overline{D}) = R_N^* = R$, therefore it is straightforward to verify that $P_S^B(\overline{D}) > P_N^B$.

the information asymmetry in the primary loan market. Specifically, expression (A) is the ratio of the expected loan quality under no information sharing to that conditional on a shared $\overline{D}$-history.[27] This ratio is smaller than 1, implying an increase in the perceived quality conditional on the borrower having no previous default. Expression (B) is the ratio of R_N^* to $R_S^*(\overline{D})$. This ratio is greater than 1, reflecting a decline in the perceived credit risk conditional on a shared $\overline{D}$-history.

The information asymmetry in both primary and secondary markets is rooted in the uncertainty of the borrower's type. These two markets, however, differ in two aspects. First, the strategic asset sale by the incumbent bank is only relevant in the secondary market. Second, the uncertainty about the aggregate state s exists only in the primary market whereas it has resolved when the secondary market opens. Such differences disappear when the parameters ρ and π approach their upper and lower limit, respectively. In particular, the strategic asset sale vanishes when $\rho = 1$. That is, it is known for sure that the incumbent bank is selling the loan not for strategic reason but because it is facing a run. In addition, the difference in the uncertainty about the state s disappears when $\pi = 0$.[28] Therefore, the primary and secondary loan markets have the same level of information asymmetry only when $\rho = 1$ and $\pi = 0$ simultaneously hold, in which case the impact of information sharing is symmetric in the two markets. As a result, the price ratio $P_N^B/P_S^B(\overline{D})$ equals 1.

The price ratio $P_N^B/P_S^B(\overline{D})$ is smaller than 1 for either $\rho < 1$, or $\pi > 0$, or both. To see so, notice that expression (A) increases in ρ. Intuitively, as the probability of a run decreases from 1, it becomes more likely that the loan is on sale for strategic reasons. As a result, the adverse selection in the secondary market aggravates, and the gap in the expected qualities widens across the two information sharing regimes, leading to a lower value for expression (A). On the other hand, expression (B) decreases in π. Intuitively, as π increases, the difference between H- and L-type borrower diminishes. The credit history becomes less relevant as an informative signal of the borrower's type, and the gap between the two equilibrium loan rates narrows, leading to a lower value of expression (B). Therefore, whenever $\rho < 1$, or $\pi > 0$, or both, information sharing's positive impact of reducing adverse selection in the secondary market dominates its negative effect of

[27] The expected quality is defined as the probability that the loan is granted to an H-type borrower.

[28] On the other hand, if $\pi = 1$, there will be no longer a difference between the H- and L- type borrowers, as both are guaranteed to succeed.

decreasing the loan rate in the primary market.

Given $P_S^B(\overline{D}) > P_N^B$, a continuity argument ensures that there must exists a set of parameters where the risk-free rate r_f lies between the two prices. In such a case, the incumbent bank that lends to an H-type borrower survives the run in state B when sharing the borrower's credit history but fails when sharing no information. We denote by $\mathbb{F}_j$ the set of parameters where inequalities $P_S^B(\overline{D}) > r_f > P_N^B$ hold in each case $j = 0, 1, 2, 3$. We establish the non-emptiness of the sets $\Psi_j \equiv \mathbb{R}_j \bigcap \mathbb{F}_j$ in the following Lemma.

Lemma 5 *When π exceeds a unique $\hat{\pi} \in (0, 1)$, there exists a non-empty set of parameters Ψ_j in Case $j = 0, 1, 2, 3$, where $P_S^B(\overline{D}) > r_f > P_N^B$ so that sharing the borrower's credit history saves the incumbent with a $\overline{D}$-loan from illiquidity. We have:*

- $\Psi_0 \equiv \mathbb{R}_0 \bigcap \mathbb{F}_0$ *with* $\mathbb{F}_0 \equiv \{(c, R)|\underline{R} < R < \overline{R}\}$.
- $\Psi_1 \equiv \mathbb{R}_1 \bigcap \mathbb{F}_1$ *with* $\mathbb{F}_1 \equiv \{(c, R)|R < \overline{R} \text{ and } c > \underline{c}\}$.
- $\Psi_2 \equiv \mathbb{R}_2 \bigcap \mathbb{F}_2$ *with* $\mathbb{F}_2 \equiv \{(c, R)|\underline{c} < c < \overline{c}\}$.
- $\Psi_3 \equiv \mathbb{R}_3 \bigcap \mathbb{F}_3$ *with* $\mathbb{F}_3 \equiv \mathbb{F}_2$.

Proof. See Appendix A.6 ■

Figure 3 gives the graphic representation of the sets Ψ_j.[29] We provide the expressions for the cutoff values (i.e., $\underline{c}, \overline{c}, \underline{R}, \overline{R}$, and $\hat{\pi}$) in Appendix A.6.

[Insert Figure 3 here]

Clearly the sets $\Psi_j, j = 0, 1, 2, 3$, represent the range of parameters where sharing borrower credit history is efficient. Moreover, each set Ψ_j contains parameters for which information sharing can endogenously emerge. Indeed, for all other parametric combinations, information sharing does not reduce the incumbent bank's liquidity risk but only induce higher competition from the entrant bank. Therefore, we focus our analysis on the sets $\Psi_j, j = 0, 1, 2, 3$, for the rest of the paper.

[29] Notice that the expressions for P_N^B and $P_S^B(\overline{D})$ are the same in Case 2 and Case 3. This is because the payoff of the loan, R, is sufficiently high that the entrant bank competes with the incumbent bank both for a loan of unknown credit history and for a loan with a $\overline{D}$-history. Therefore, we have $\mathbb{F}_3 = \mathbb{F}_2$.

Ex-ante decision on sharing credit history.

We are now in position to determine whether the incumbent bank voluntarily chooses to share the borrower's credit history at $t = 0$. Let us denote with V_i the incumbent bank's expected profits at $t = 0$ under the information sharing regime $i \in \{N, S\}$. We denote by φ_j the set of parameters where $V_S > V_N$ in Case $j = 0, 1, 2, 3$. The next proposition establishes the non-emptiness of the sets φ_j.

Proposition 2 *The incumbent bank voluntarily chooses to share its borrower's credit history in region $\varphi_j = \Psi_j$ for Case $j = 0, 3$, and in region $\varphi_j \subseteq \Psi_j$ for Case $j = 1, 2$. We have $\varphi_j = \Psi_j$ for Case $j = 0, 1, 2, 3$, if and only if $\rho \geq \overline{\rho} \equiv (1-\alpha)(1-\delta)$, so that the incumbent bank voluntarily discloses such information when information sharing is efficient.*

Proof. See Appendix A.7. ■

To illustrate the intuition of the result, let us decompose the difference between the incumbent bank's expected profits in the two information sharing regimes as follows:

$$V_S - V_N = \underbrace{[\alpha + (1-\alpha)\delta\pi](R_S^*(\overline{D}) - R_N^*)}_{(1)} + \underbrace{(1-\alpha)(1-\delta)\pi(R_S^*(D) - R_N^*)}_{(2)} + \underbrace{\alpha(1-\pi)\rho R_N^*}_{(3)}.$$

Term (1) represents the *competition* effect. It is non-positive because $R_S^*(\overline{D}) \leq R_N^*$. Since disclosing the borrower's credit history $\overline{D}$ encourages the entrant bank to compete for the borrower, the incumbent bank can lose its market power by engaging in information sharing. Term (2) is understood in the literature as the *capturing* effect. It is non-negative as $R_S^*(D) \geq R_N^*$. Sharing information about the borrower with a default history can deter entry, so that the incumbent bank can capture such borrower and charge a higher loan rate. Finally, Term (3) is positive and denotes the new effect that our model features. We refer to it as the *liquidity* effect and it highlights the extra benefit of sharing information. Revealing the borrower having no previous default reduces the adverse selection in the secondary market, so that the incumbent bank will be saved from a run in state B.

The incumbent bank voluntarily engages in information sharing if and only if $V_S - V_N > 0$. In Case 0 and Case 3, credit information sharing always dominates, so that we have $\varphi_j = \Psi_j$ for $j = 0, 3$. The reason is that the incumbent bank faces no cost to share information in either case. In Case 0, the credit market features no contestability. The entrant bank never competes for the borrower—independent of the information sharing regime, i.e., $R_S^*(\overline{D}) = R_N^* = R_S^*(D) = R$. As a result, only the liquidity effect is present.

On the other hand, the credit market is perfectly contestable in Case 3. The entrant bank always competes for the borrower—independent of the information sharing regime. The incumbent bank avoids inefficient liquidation, but only earns an expected profit equal to the entrant bank's entry cost, that is $V_S = c$. Whereas without information sharing, the incumbent bank fails in state B when experiencing a run—even if the bank holds an H-type loan. As a result, the expected profit under no information sharing is $V_N = c - \alpha(1-\pi)\rho R_N^* < V_S$. The expression $\alpha(1-\pi)\rho R_N^*$ reflects the expected liquidation loss.

The incumbent bank does incur a cost for sharing the borrower's credit history in Case 1 and Case 2, as reflected by $R_S^*(\overline{D}) < R_N^*$. When the probability of runs is sufficiently small (that is, $\rho < \overline{\rho}$) the cost of information sharing dominates its benefit. As a result, the set of parameters where information sharing is privately optimal is a subset of the set where information sharing is efficient. That is, $\varphi_j \subset \Psi_j$ for $j = 1, 2$. The sets φ_j would still be non-empty for $\rho \in (0, \overline{\rho})$ though. For example, near the lower bound of Case 1 (that is, R marginally higher than $R_S^E(\overline{D})$), the competition effect is close to zero; the capturing effect is zero, but the liquidity effect remains $\alpha(1-\pi)\rho R$, which makes $V_S > V_N$. Similarly, near the upper bound of Case 2 (that is, R marginally lower than $R_S^E(D)$), one can verify that the competition and capturing effects mostly cancel, so that $V_S > V_N$ is again driven by the liquidity effect. Finally, when the probability of a run is sufficiently high (that is, $\rho \geq \overline{\rho}$) the benefit from information sharing would still dominate the cost, such that $\varphi_j = \Psi_j$ also for $j = 1, 2$. Figure 4 graphically summarizes the results.

[Insert Figure 4 here]

Our results, as already mentioned, can also be understood from the perspective of the Bayesian persuasion literature. The incumbent bank can be considered as a persuader and the asset buyers as (homogenous) receivers. By choosing the information sharing regime, the incumbent bank commits to a test on the quality of the loan it holds. When making the decision about sharing information, the incumbent bank has the same prior as the asset buyers. Without information sharing, the asset buyers price the loan on sale according to the prior, taking into account the adverse selection. In regions φ, buyers bid a price $P_N^B < r_f$, which would lead to inefficiently liquidation of an H-type loan. The incumbent bank can increase its expected payoff by generating a binary signal $x \in \{\overline{D}, D\}$ by committing to sharing the borrower's credit history. Consequently, the buyers' beliefs about the loan quality improves when the signal is $\overline{D}$ and deteriorates otherwise. Given the posterior beliefs, the secondary-market loan prices are such that $P_S^B(D) < P_N^B < P_S^B(\overline{D})$. When state B realizes, the incumbent bank receives a zero payoff in region φ, both when the information sharing generates a signal D and under the no information sharing regime. However, when the information sharing generates a signal $\overline{D}$, the incumbent bank boosts the price of the loan in the secondary market in state B, avoiding the inefficient liquidation of the H-type loan. As a result, the incumbent bank has incentives to persuade the buyers.[30]

3.3 Sharing Unverifiable Credit History

We now relax the assumption on the verifiability of credit history. If the reported borrower's credit history is not verifiable, the incumbent bank may have an incentive to overstate the borrower's past credit performance. In particular, we consider the possibility to misrepresent a borrower who has previously defaulted as one with no default history. A pre-requisite for the incumbent bank to manipulate the reported credit history is that the bank must have chosen the information sharing regime in the first place. Therefore, we restrict our analysis to the sets of parameters φ_j with $j = 0, 1, 2, 3$, as defined in Section 3.2.

The incumbent bank faces two considerations when it has to decide whether to misreport a D-history. On the one hand, by overstating the past credit performance, the bank will be able to boost its secondary-market loan price and survives a run in state B. Therefore, the expected gain of lying is

$$(1-\pi)P_S^B\left(\overline{D}\right).$$

On the other hand, upon the (false) announcement of a $\overline{D}$ credit history, the entrant bank will compete more fiercely for the borrower so the incumbent bank can charge a lower rate in the primary loan market. Therefore, the expected loss of lying is

$$\pi\left[R_S^*(D) - R_S^*(\overline{D})\right].$$

[30] As Kamenica and Gentzkow (2011) points out, Bayesian rationality imposes only one constraint on receivers' posterior beliefs. That is, their posterior beliefs must be Bayes-plausible in equilibrium. It can be verified that our model satisfies this requirement, because $Pr(H|\overline{D})Pr(\overline{D}) + Pr(H|D)Pr(D) = \alpha$ and $Pr(L|\overline{D})Pr(\overline{D}) + Pr(L|D)Pr(D) = 1 - \alpha$.

A necessary condition for the incumbent bank to truthfully reveal the borrower's past default is that $R_S^*(D) > R_S^*(\overline{D})$. Otherwise, the incumbent bank will only have incentive to lie. By implication, the incumbent bank will never have the incentive to truthfully reveal the borrower's credit history in region φ_0 where $R_S^*(D) = R_S^*(\overline{D})$.

In the other regions (i.e., φ_1, φ_2 and φ_3) the loan market is more contestable and $R_S^*(D) > R_S^*(\overline{D})$, so that the expected benefit from truth-telling is positive. However, even if ex-post the incumbent bank has an incentive to truthfully communicate the credit history, it is possible that ex-ante the bank is unwilling to share such information. For truthful information sharing to be voluntary, the set of parameters that guarantees truth-telling has to overlap with the set of parameters that makes information sharing ex-ante profitable. In other words, the ex-ante incentive constraint for voluntary information sharing has to be simultaneously satisfied with the ex-post incentive constraint for truth-telling.

We show that the incumbent bank's incentive to truthfully disclose the borrower's credit history is positively related to the contestability of the primary loan market. Truth-telling cannot be sustained in region φ_1. In this case, the loan rate $R_S^*(D)$ is bounded above by the loan's relatively low return R, so that the expected loss of lying is limited and the expected gain dominates. In the regions φ_2 and φ_3, the return R becomes larger and the expected loss of a deviation from the equilibrium can dominate the benefit. Truthful reporting of credit history therefore becomes sustainable. The following proposition reports the results on truthful revelation.

Proposition 3 *The incumbent bank truthfully discloses the borrower's credit history only if $R_S^*(D) > R_S^*(\overline{D})$. Truthful communication of credit history cannot be sustained in regions φ_0 and φ_1, and it is sustainable in the whole region φ_3. In region φ_2, there exists a set of parameters $\varphi_2' \subseteq \varphi_2$ where truth-telling is sustainable. Furthermore, there exists a unique $\underline{\rho} \in (0, \overline{\rho})$ such that $\varphi_2' = \varphi_2$ when $\rho < \underline{\rho}$.*

Proof. See Appendix A.8 ■

The results on truthful disclosure are illustrated in Figure 5. As compared to to Figure 4, it highlights a dark-blue area corresponding to the set of parameters in which truth-telling can be sustained. Truth-telling cannot be sustained in Case 0 and Case 1, but can be sustained in Case 3 whenever the bank finds it profitable to share information at $t = 0$. In Case 2, we depict a scenario where $\underline{\rho} < \rho < \overline{\rho}$, so that truth-telling is sustained only in a subset of φ_2. In Figure 5, this subset is depicted by the area above the line R_T, a cutoff value above which truthful information sharing can be sustained.

[Insert Figure 5 here]

4 Policy Implications

From our discussion in Section 3.2, it should be clear that the incumbent bank's private decision on information sharing is not always socially efficient. In particular, when $\rho < \overline{\rho}$, in both Case 1 and Case 2 there exists a region where the incumbent bank chooses not to engage in information sharing—even though doing so can save the bank from runs in state B.[31] That is, when the probability of liquidity risk is small, the incumbent bank finds it privately optimal to be exposed to that risk. It is (privately) too costly to give up the position as an information monopoly in all states, to benefit from the boosted asset market liquidity only with a small probability.[32]

The imperfectly aligned incentives (reflected by $\varphi_j \subset \Psi_j$ for $j = 1, 2$) leave scope for policy intervention. We now consider the efficiency implication of establishing a public credit registry, by which a regulator commands the incumbent bank to share the borrower's credit history. Compared to the private institution, the regulator has a different preference for information sharing because she is only concerned with the inefficient liquidation of the incumbent bank but not its loss of monopolistic rent. Such a loss only constitutes a transfer from the incumbent bank to the borrower and is therefore efficiency neutral. The point can be illustrated by comparing Figure 3 and 4. Within the regions Ψ_1 and Ψ_2 where information sharing can save the incumbent bank from illiquidity, the incumbent bank finds it too costly to share the borrower's credit history in the area between R_1 and R_2.[33] Therefore, when the shared credit history is verifiable, imposing a public registry

[31] On the other hand, when $\rho \geq \overline{\rho}$, the private decision on information sharing is also socially efficient in regions Ψ_1 and Ψ_2. The private decision is always efficient in regions Ψ_0 and Ψ_3.

[32] Notice that even if it is feasible to share borrower's type, as assumed in Section 3.1, the inefficiency would remain. Indeed, it would be efficient to share borrower's type to eliminate the adverse selection in the secondary market (so to avoid the costly liquidation of an H-type loan). However, according to Proposition 1 the incumbent bank finds it profitable to share information only if $\rho > \hat{\rho}$. When $\rho < \hat{\rho}$, we have an inefficient decision similar to the case of sharing information on the borrower's credit history.

[33] The definitions of R_1 and R_2 can be found in the legend to Figure 4 and the proof of Proposition 2.

would improve efficiency in the area. When the shared credit history is not verifiable, the policy intervention also needs to make sure that the bank truthfully reports to the public credit registry. Imposing a public registry would only improve efficiency in regions where the incumbent bank finds it ex ante unprofitable to share the borrower's credit history but ex post has incentives to truthfully disclose such information. We show that a public registry can improve efficiency under the incumbent bank's ex-post incentive compatibility constraint for truth-telling. The range of parameters where a public registry generates efficiency gain with non-verifiable credit history is illustrated in Figure 6.

[Insert Figure 6 here]

In particular, it is represented by the area below R_2 (so that the bank finds it too costly to share information) and above R_T (so that ex post there is incentive for truthful disclosure). Such a region is non-empty when $\rho < \underline{\rho}$. The following corollary summarizes the policy implications of our model.

Corollary 1 *A public registry can improve efficiency only if* $\rho < \overline{\rho}$.

- *When the credit history is verifiable, the public registry improves efficiency in region* $\{(c, R)|R_1 < R < R_2\} \cap \Psi_j \neq \varnothing$, $j = \{1, 2\}$.
- *When the credit history is not verifiable, the public registry improves efficiency if and only if* $\rho < \underline{\rho}$. *In particular, the efficiency gain is obtained in region* $\{(c, R)|R_T < R < R_2\} \cap \Psi_2 \neq \varnothing$. *Furthermore, there exists a unique* $\underline{\rho}' \in (0, \underline{\rho})$, *such that for* $\rho < \underline{\rho}'$ *the efficiency gain is also obtained in region* $\{(c, R)|R_T < R < R_2\} \cap \Psi_1 \neq \varnothing$.

Proof. See Appendix A.9 ■

5 Robustness and Discussions

We now check the robustness of our main assumptions and discuss possible extensions of the model.

The existence of such area is guaranteed by the condition $\rho < \overline{\rho}$.

5.1 The possibility of sharing borrower types

For the main analysis of the paper, we focus on the sharing of credit history. While the choice is motivated by empirical observations, from a theoretical point of view, the question remains whether it would be an optimal choice of the incumbent bank. In particular, if the bank can communicate directly the information about the borrower's creditworthiness (e.g., the borrower's type in our model), would the bank find it profitable to do so?

We show now that sharing only the borrower's credit history is indeed preferred to directly sharing the borrower's type. The difference between the expected payoff of sharing the credit history and that of sharing the type can be written as follows:

$$V_S - V_S^{Type} = \underbrace{\alpha\left[R_S^*(\overline{D}) - R_S^*(H)\right]}_{\text{(1) Competition effect}} + \underbrace{(1-\alpha)\delta\pi\left[R_S^*(\overline{D}) - R_S^*(L)\right]}_{\text{(2) Capturing effect}}.$$

Whenever sharing the credit history saves the incumbent bank from runs, sharing the type does not generate extra benefit via the improved market liquidity of the loan. This is true both from a private and a social point of view. Sharing types therefore only affects bank's payoffs via the changes of the loan rates. When the information on the borrower's type is communicated, an H-type borrower will no longer be pooled with an L-type borrower with a $\overline{D}$-history. Accordingly, the difference in the expected profits can be decomposed into two components. On the one hand, the incumbent bank has to charge a lower loan rate for an H-type borrower (i.e., $R_S^*(\overline{D}) - R_S^*(H) > 0$). On the other hand, the bank can charge a higher loan rate for an L-type borrower (i.e., $R_S^*(\overline{D}) - R_S^*(L) < 0$).[34] We indicate these two components again as *competition* effect and *capturing* effect, respectively.

We show that $V_S - V_S^{Type} \geq 0$, so that the competition effect always dominates. In fact, $V_S = V_S^{Type} = c$ when the credit market is perfectly contestable (i.e., $R > R_S^E(L)$), and the incumbent bank earns a profit equal to the entrant bank's entry cost, regardless of the information shared. For a lower return $R \in \left(R_S^E(\overline{D}), R_S^E(L)\right)$, the loss of rent on an L-type borrower is more limited, whereas the magnitude of the competition effect remains the same, making the sum of the two effects positive. When R further drops below $R_S^E(\overline{D})$, the capturing effect becomes zero, whereas the competition effect remains positive. The following corollary summarizes this result.

[34] Recall that condition (3) guarantees that an L-type loan has a positive NPV.

Corollary 2 *When sharing credit history can save the incumbent bank of an H-type loan from runs, the incumbent bank prefers sharing the borrower's credit history to sharing types. In particular, $V_S > V_S^{Type}$ in region Ψ_j, with $j = 0, 1, 2$, and $V_S = V_S^{Type}$ in region Ψ_3.*

5.2 Unfairly priced deposits

To isolate the information asymmetry problem, we have assumed deposits to be fairly priced so that risk-shifting incentives due to limited liability do not arise. When deposits are insured and unfairly priced, the incumbent bank's incentives to share credit information would be lower. However, there still exist regions where voluntary information sharing will endogenously emerge.

Since the secondary-market price for an H-type loan is lower without information sharing than with information sharing, choosing the regime of no sharing information allows the incumbent bank to enjoy higher market power at the cost of higher liquidity risk. Unfairly priced deposits give the bank incentives to engage in such risk-taking. Let us show why this is the case by considering the simplest situation where the deposit insurance is provided to banks free of charge and the incumbent bank can share verifiable information on the borrower's type. As an example, let us consider Case 1 analyzed in Appendix A.2 (i.e., the proof of Proposition 1). Let V' denotes the incumbent bank's expected profit at $t = 0$ when there is free deposit insurance. For $r_f \in \left(P_N^B, P_S^B(H)\right)$, the incumbent bank's expected profit with and without information sharing can be expressed as $V_S' = \alpha(c + r_f - r_f) + (1 - \alpha)\pi(R - r_f)$ and $V_N' = [\pi + (1 - \pi)\alpha(1 - \rho)](R - r_f)$, respectively. On the other hand, recall from Section 3.1 that the expected profits with fairly priced deposits are $V_N = [\pi + (1 - \pi)\alpha(1 - \rho)]R - r_f$ and $V_S = \alpha(c + r_f) + (1 - \alpha)\pi R - r_f$, respectively. Thus, we obtain

$$(V_S - V_N) - (V_S' - V_N') = \alpha(1 - \pi)\rho \cdot r_f,$$

where the positive difference reflects the subsidy of the deposit insurance when the bank fails in state B in a run. Notice that the bank is compensated for r_f for its total loss of R in that state. This decreases the liquidation cost and the bank's incentive to share information.[35] However, one can still show the existence of a $\hat{\rho}' \in (\hat{\rho}, 1)$ such that $V_S' > V_N'$ and information sharing endogenously emerges.

[35] A similar result holds also in Case 2. In Case 0 and Case 3, it still holds that the incumbent bank always chooses to share information.

5.3 Risky H-type borrowers

Instead of assuming the H-type borrower to be risk-free, we now relax the assumption to allow an H-type borrower only to succeed with probability $q \in (0,1)$ in state B. We show that the main trade-off between secondary-market liquidity and primary-market rent extraction remains. For simplicity, we analyze a case where the incumbent bank can share verifiable information about the borrower's type. The result can be extended to the case where the incumbent bank shares the borrower's credit history.

In the presence of information sharing, the price of an H-type loan will be $P_S^B(H) = qR_S^*(H)$ in the state B, and $P_S^G(H) = R_S^*(H)$ in the state G. The price of an L-type loan remains $P_S^B(L) = 0$ and $P_S^G(L) = R_S^*(L)$, in the state B and G, respectively. We now move one step back and consider the incumbent bank's funding. The deposit rate to finance an L-type loan remains equal to r_f/π. However, compared to the setup where $q = 1$, the depositors' break-even rate for an H-type loan increases from r_f to $r_S^I(H) = r_f/[\pi + (1-\pi)q]$, reflecting the fact that the loan can now default in the state B with probability $1-q$. Moving another step back, one can show that the entrant bank's break-even loan rate is $R_S^E(H) = (c+r_f)/[\pi + (1-\pi)q]$ for an H-type borrower and $R_S^E(L) = (c+r_f)/\pi$ for an L-type borrower. As a result, the equilibrium loan rate will be $R_S^*(H) = \min\left\{R_S^E(H), R\right\}$ and $R_S^*(L) = \min\left\{R_S^E(L), R\right\}$ for the H- and L-type borrower, respectively.

With information sharing, the equilibrium secondary market asset price for an H-type loan can be expressed as

$$\begin{aligned} P_S^B(H) &= q \cdot \min\left\{R_S^E(H), R\right\} \quad &\text{if} \quad s = B, \\ P_S^G(H) &= \min\{R_S^E(L), R\} \quad &\text{if} \quad s = G. \end{aligned}$$

Following Appendix A.1 (i.e., the proof of Lemma 1), the equilibrium secondary-market loan price without information sharing are

$$\begin{aligned} P_N^B &= \frac{q \cdot \alpha\rho}{(1-\alpha)+\alpha\rho} \min\left\{R_N^E, R\right\} \quad &\text{if} \quad s = B, \\ P_N^G &= \min\{R_N^E, R\} \quad &\text{if} \quad s = G, \end{aligned}$$

where the entrant bank's break-even loan rate $R_N^E = (c+r_f)/[\alpha(\pi+(1-\pi)q)+(1-\alpha)\pi]$.

It is straightforward to establish that $R_S^*(H) \leq R_N^*$, so that the incumbent bank does lose rent from intensified primary loan market competition.[36] Furthermore, $P_N^B < P_S^B(H)$

[36] The inequality is strict when both loan rates are interior solutions.

is always true, which indicates that information sharing boosts the secondary-market price for bank's loan. There exists r_f such that $P_N^B < r_f$ and $P_S^B(H) > r_S^I(H)$, such that the incumbent bank is saved from runs in state B under the information sharing regime and it fails under the no information sharing regime.

The incumbent bank still trades off the benefit of increasing secondary-market liquidity against the loss of monopolistic rent when making ex-ante information sharing decisions. Thus, introducing a risky H-type borrower will not qualitatively change our main results, and a result similar to Proposition 1 would hold.

5.4 Mutual information sharing

We discuss now how to introduce into our model the mutual information sharing agreement à la Pagano and Jappelli (1993). In spirit of this classic model, we assume two towns, each of which has a bank and a borrower who needs 1 unit of funding. In each town, the bank has an ongoing lending relationship with the borrower. At $t = 0$, both banks decide whether to enter into a mutual information sharing agreement or not. At $t = 1$, a bank shares the information about the borrower's type or previous credit history if it has chosen the information sharing regime, and discloses no information otherwise. At $t = 2$, banks compete for borrowers in both towns, and the winner issues deposits to finance the loan(s). At $t = 3$, both banks could undergo liquidity shocks and need to conduct a loan sale on the secondary market to asset buyers as in our model.

With the assumption that a bank needs to pay an additional cost c to extend credit to the borrower who lives in the other town,[37] there would exist an equilibrium where the local bank always wins the local borrower. While continuing to stay with the incumbent bank, the borrower benefits from the lower loan rate under information sharing thanks to the intensified primary market competition.

It is straightforward to see that our analysis extends to this symmetric setup. When it is profitable to share information (that is, when $V_S > V_N$) an incumbent bank will do so, regardless of the competing bank's decision on information sharing decision. In other words, information sharing is the incumbent bank's dominant strategy when $V_S > V_N$, so that our results are robust to a symmetric setup in which information sharing is the outcome of a mutual agreement.

[37] The original setup of Pagano and Jappelli (1993) also features the same assumption.

5.5 Diversification and loan portfolio risk

In our analysis, we worked with a one-loan setup for its tractability. Our results should be robust to the diversification of idiosyncratic risks. As long as there remains uncertainty on the asset quality (e.g., driven by a systematic risk) and the scope for adverse selection, our results will continue to hold. Indeed, the model can be recast into a setup where the incumbent bank has private information on the quality of its loan portfolio. We consider the following setting where the incumbent bank inherits a unit loan portfolio consisting of a fraction ω of H-type loans and a fraction $1-\omega$ of L-type loans. There is uncertainty about the portfolio composition, that is $\omega \in \{\omega_H, \omega_L\}$ with $\omega_H > \omega_L$. The prior of holding a H-portfolio (that is, a portfolio with a fraction ω_H of H-type loans) is $Pr(\omega = \omega_H) = \gamma \in (0,1)$. For illustrative purpose, we examine again the simplest case in which the incumbent bank shares verifiable information about the borrowers' type.

We show in this alternative setup that information sharing increases the price of the bank's loan portfolio in the secondary market. Allowing partial liquidation of the loan portfolio, let us first analyze a case where the incumbent bank can commit to selling its L- and H-type loans according to their proportion in the portfolio. Let P_N^B be the price for the incumbent's loan portfolio in state B without information sharing. We have

$$P_N^B = \left(\frac{\rho\gamma}{\rho\gamma + (1-\gamma)}\omega_H + \frac{1-\rho\gamma}{\rho\gamma + (1-\gamma)}\omega_L\right) R_N^*$$

where

$$R_N^* = \min\left\{R, \left[\frac{\gamma}{\omega_H + (1-\omega_H)\pi} + \frac{1-\gamma}{\omega_L + (1-\omega_L)\pi}\right](c + r_f)\right\}$$

is determined either by the entrant bank's break-even condition or R. Note that $\omega_H R_N^*$ and $\omega_L R_N^*$ are the returns of the H- and L-portfolio in state B, respectively. With $P_N^B < \omega_H R_N^*$, the incumbent bank sells its high quality loan portfolio only if it faces the liquidity shock.

When the information on borrowers' type is shared, the same liquidation strategy leads to a price $P_S^B(HP) = \omega_H R_S^*(H)$ and $P_S^B(LP) = \omega_L R_S^*(H)$ for an H- and L-portfolio, respectively. The equilibrium loan rates $R_S^*(H)$ and $R_S^*(L)$ are the same as in Section 3.1. Following the proof of Proposition 1 (see Appendix A.2), one can show that $P_S^B(HP) > P_N^B$. That is, information sharing boosts the secondary-market price of the loan portfolio. For $r_f \in \left(P_N^B, P_S^B(HP)\right)$, the incumbent bank with an high quality portfolio will fail without information sharing, while the bank can sell a fraction $\beta = r_f / P_S^B(HP)$ to meet the depositors' withdrawal r_f with information sharing.

Finally, if the bank cannot commit to selling its L- and H-type loans by their proportion in the portfolio (i.e., the bank always sells L-type loans first), the adverse selection problem aggravates and the loan price drops further without information sharing. As a result, the above results will continue to hold.

5.6 Securitization

Our results should be robust to the assumption that the incumbent bank raises liquidity through securitization instead of the direct sale of its loan. The direct asset sale is equivalent to issue 100% equity against the underlying asset. Securitization, on the other hand, allows the bank to tranche the cash flow and issue securities with different seniorities backed by the same underlying asset. In the same manner that sharing no credit information increases the bank's funding liquidity risk, the lack of information on the underlying loan quality would entail higher private credit enhancement for the resulting (senior) securities to be attractive to outside investors. Therefore, the bank may voluntarily choose to share credit information to reduce the cost of providing private credit enhancement.

5.7 Endogenous liquidity risk

We interpret the incumbent bank's liquidity risk as the risk of bank runs. Runs are assumed to occur with an exogenous probability ρ, independently of the bank's fundamental, i.e. the type of loans and the state s. In other words, runs are modeled as sun-spot events. In contrast, models based on global games link bank's liquidity risk to fundamentals. In global games, bank run games typically feature a threshold equilibrium. Runs occur if and only if the bank's realized fundamental is lower than a critical threshold, which usually decreases in the liquidation value of banks' assets. Our main results should be robust to this alternative setting.

As illustrated in our model, the price of the bank's loan on sale increases with information sharing, because adverse selection mitigates in the secondary market. Accordingly, the depositors are reassured that the liquidation loss would not be as high as in the case without information sharing. They will, therefore, have lower incentives to run on the bank. Information sharing, on the one hand, reduces the probability of bank failure, and, on the other hand, undermines the bank's position as an information monopoly. The bank may still optimally commit to information sharing, as it trades off between higher probability of survival and lower expected profit conditional on survival.

6 Conclusions

This paper formally analyzes the conjecture that banks share their borrowers' credit information to maintain the market liquidity of their loans. Information friction in the secondary loan markets makes asset sale costly and high-quality loans can be priced below their fundamental value. This basic observation implies that banks could find it convenient to disclose their borrowers' credit information to reduce the information asymmetry about the quality of their loans. Information sharing can make banks more resilient to funding liquidity risks. For credit information sharing to be a private optimal choice of banks, the benefit of higher secondary-market liquidity has to be traded off against potential losses of market power in primary loan markets. We characterize conditions under which information sharing is efficient, privately profitable, and robust to incentives for misreporting. We also provide a rationale for the establishment of public credit registries.

Our model should be interpreted with two caveats. First, historically, governments' goal in creating public credit registries has been to improve SMEs' access to financing in primary loan markets. Our theory does not deny this benefit but shows that an overlooked benefit of information sharing is the development of secondary markets. Second, it is not our intention to claim that information sharing is the main reason for the explosion of the markets for asset-backed securities. It is ultimately an empirical question to what extent information sharing has fueled such markets expansion.

Our theoretical exposition also opens road for future empirical research. The model implies that information sharing will facilitate banks' liquidity management and loan sale. Moreover, the model suggests that information sharing system can be more easily established and work more effectively in countries with competitive banking sector, and in credit market segments where competition is strong.

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Appendix: Proofs

.1 Proof of Lemma 1

When the incumbent bank does not share any information, the game features incomplete information. Therefore, we apply the solution concept of PBE. We show that the unique pure-strategy equilibrium is a pooling equilibrium, as the incumbent bank offers a unified loan rate $R_N^I \in [r_f, R]$ and a unified deposit rate $r_N^I \in [r_f, r_f/\pi]$, independent of the type of borrower it finances.[38]

Definition 1. *A pure-strategy pooling PBE of the game g_N is characterized as follows.*
(i) An equilibrium strategy profile. Based on its knowledge of the borrower's type, the incumbent bank at $t = 2$ sets a loan rate R_N^I for the borrower and offers a take-it-or-leave-it deposit rate r_N^I to depositors. When having financed the borrower, the incumbent bank decides at $t = 3$ whether to sell the loan, according to the loan quality, the state s, and its own liquidity position. The entrant bank offers a competing loan rate R^E without knowing the borrower's type or credit history. Depositors choose to provide funding or not based on the offered deposit rate. Asset buyers bid P_N^G in state G and P_N^B in state B to purchase any loan on sale.
(ii) A system of beliefs. The entrant bank holds the prior belief about the borrower's type. The depositors Bayesian update their beliefs according to the deposit rate offered by the incumbent bank. The asset buyers Bayesian update their beliefs according to the aggregate state s and observation of asset sale.
(iii) The strategy profile in (i) is sequential rational given the beliefs in (ii).

The proof consists of two parts. In Part I, we show that the strategy profile and belief system described in Lemma 1 indeed constitute a PBE. In Part II, we prove that no other pure-strategy PBE exists.

Part I: The existence of a pure-strategy pooling PBE.

To establish the equilibrium described in Lemma 1, we solve the game backwards.

Step 1. We start by analyzing the secondary loan market in the state G and B respectively. In state G, the incumbent bank sells its loan only when facing a run, and in that

[38] Notice that the incumbent bank holds private information about the borrower's type as well as the credit history. Note that the borrower's type is more relevant than the credit history. Therefore, without losing generality, we consider the incumbent bank directly conditions its loan rates on the true types.

case sells its loan regardless of the borrower's type. The asset buyers, therefore, believe the loan on sale to be an H-type with probability α and an L-type with probability $1-\alpha$. Moreover, both H- and L-type borrowers will repay R_N^* in state G. As a result, asset buyers' competitive bidding leads to the following break-even condition

$$\alpha(R_N^* - P_N^G) + (1-\alpha)(R_N^* - P_N^G) = 0,$$

which implies

$$P_N^G = R_N^*. \tag{7}$$

In state B, the incumbent bank always sells its L-type loan and sells an H-type loan only if facing a run, and asset buyers Bayesian update their belief accordingly. That is,

$$Prob(H|\text{loan sale}) = \frac{Pr(\text{run})Pr(H)}{Pr(\text{run})Pr(H) + Pr(L)} = \frac{\alpha\rho}{\alpha\rho + (1-\alpha)}.$$

Since only the H-type borrower will replay R_N^* in state B, the asset buyers' competitive bidding leads to the following break-even condition

$$\frac{\alpha\rho}{\alpha\rho + (1-\alpha)}\left(R_N^* - P_N^B\right) + \frac{1-\alpha}{\alpha\rho + (1-\alpha)}\left(0 - P_N^B\right) = 0,$$

which implies

$$P_N^B = \frac{\alpha\rho}{(1-\alpha) + \alpha\rho} R_N^* < R_N^*. \tag{8}$$

One can verify that the incumbent bank's equilibrium strategy is sequentially rational given the asset buyers' beliefs and bids. In state B, an L-type loan is doomed to fail, so the incumbent bank optimally sells an L-type loan for $P_N^B > 0$. If the loan is an H-type, the incumbent bank optimally holds it to maturity unless it experiences runs, because $P_N^B < R_N^*$. In state G, the incumbent bank is indifferent between holding the loan to maturity and selling the loan in the secondary market when facing no run, as both actions generate the same revenue R_N^*. Given an innocuous assumption that the incumbent bank holds its loan to maturity when facing no run in state G, asset buyers' belief remains the same as the prior, which is consistent with the incumbent bank's strategy.

Step 2: We now move to the stage where the incumbent bank raises its funding. The strategic interaction between the incumbent bank and its depositors can be considered as a signaling game. The incumbent bank (sender) has private information about its loan quality and offers a deposit rate (a message) to the depositors (receivers). The depositors may infer the bank's loan type when deciding on accepting the bank's offer or not. The bank raises 1 unit of funding from depositors if its offer is accepted, and receives a zero payoff otherwise, as in the latter case no loan will be financed.

We first analyze depositors' belief and strategy on the equilibrium path described by Lemma 1. As the incumbent bank offers a pooling deposit rate r_N^I, the depositors belief about the bank's loan quality remains the same as the prior. That is,

$$Pr(H|r_N^I) = Pr(H) = \alpha, \quad Pr(L|r_N^I) = Pr(L) = 1 - \alpha.$$

Furthermore, under the condition

$$P_N^B = \frac{\alpha\rho}{\alpha\rho + (1-\alpha)} R_N^* < r_f,$$

the incumbent bank cannot raise enough liquidity from the loan sale when a run happens. The depositors therefore anticipate the incumbent bank to fully repay its debt in state G and to be able to repay in state B only when it holds an H-type of loan *and* faces no run. Given the depositors' belief about the loan quality, the minimum rate that depositors are willing to accept is equal to

$$r_N^I = \frac{r_f}{\pi + (1-\pi)\alpha(1-\rho)}. \tag{9}$$

This rate r_N^I allows depositors to break even given the subsequent equilibrium strategies of the incumbent bank and asset buyers, that is

$$Pr(G)r_N^I + Pr(B)Pr(H|r_N^I)Pr(\text{no run})r_N^I = [\pi + (1-\pi)\alpha(1-\rho)]r_N^I = r_f.$$

The incumbent bank's equilibrium deposit rate r_N^I can be sustained by depositors' off-equilibrium belief $Pr(H|r^I \neq r_N^I) < \alpha$ and $Pr(L|r^I \neq r_N^I) > 1-\alpha$. In other words, depositors hold a worse-than-prior belief about the incumbent bank's loan quality when receiving an off-equilibrium deposit rate $r^I \neq r_N^I$. Given the off-equilibrium beliefs, depositors' break-even deposit rate can be computed as

$$\hat{r} = \frac{r_f}{\pi + (1-\pi)Pr(H|r^I \neq r_N^I)(1-\rho)} > r_N^I.$$

Given the depositors' break-even rate, it is indeed sequentially rational for the incumbent bank to offer the pooling deposit rate r_N^I. In particular, when the incumbent bank wins an H-type loan, it earns an expected profit

$$\Pi_N(H) = \pi(R_N^* - r_N^I) + (1-\pi)(1-\rho)(R_N^* - r_N^I) = [\pi + (1-\pi)(1-\rho)](R_N^* - r_N^I) > 0$$

by offering the equilibrium deposit rate r_N^I.[39] Suppose, instead, the incumbent bank deviates by offering a deposit rate $r^I \neq r_N^I$, then its expected profit is either

$$\hat{\Pi}_N(H) = 0 < \Pi_N(H) \quad \text{if} \quad r^I < \hat{r},$$

or

$$\hat{\Pi}_N(H) = [\pi + (1-\pi)(1-\rho)](R_N^* - \hat{r}) < \Pi_N(H) \quad \text{if} \quad r^I > \hat{r}.$$

Thus, the incumbent bank has no profitable deviation by offering a deposit rate $r^I \neq r_N^I$.

Similarly, when the incumbent bank wins an L-type loan, it earns an expected profit

$$\Pi_N(L) = \pi(R_N^* - r_N^I) > 0$$

by offering the equilibrium rate r_N^I. While its expected profit from deviation is either

$$\hat{\Pi}_N(L) = 0 < \Pi_N(L) \quad \text{if} \quad r^I < \hat{r},$$

or

$$\hat{\Pi}_N(L) = \pi(R_N^* - \hat{r}) < \Pi_N(L) \quad \text{if} \quad r^I > \hat{r}.$$

Therefore, the incumbent bank has no profitable deviation when holding an L-type loan either. In fact, the worse-than-prior off-equilibrium belief is a necessary and sufficient condition for r_N^I to be part of the PBE.[40]

Step 3: We now analyze the primary loan market competition between the incumbent and entrant banks.

[39] When $R_N^* = R$, the inequlity $R_N^* > r_N^I$ is guaranteed by condition (3). When $R_N^* = R_N^E = \frac{c+r_f}{\alpha+(1-\alpha)\pi}$, the inequality is guaranteed by assumption (2).

[40] The worse-than-prior off-equilibrium belief is also a necessary condition. If the off-equilibrium beliefs are more optimistic than the prior, the incumbent bank will have the incentive to deviate from offering r_N^I. Therefore, a pooling equilibrium that features the deposit rate r_N^I must be associated with worse-than-prior off-equilibrium beliefs.

Given the entrant bank's belief $Prob(H) = \alpha$ and $Prob(L) = 1 - \alpha$, the minimum loan rate that satisfies the participation constraint of the entrant bank is

$$R_N^E = \frac{c + r_f}{\alpha + (1-\alpha)\pi}. \tag{10}$$

Otherwise, the entrant bank will be better off holding the risk-free asset. That is,

$$[\alpha + (1-\alpha)\pi]\, R^E - c < r_f,\ \forall R^E < R_N^E.$$

We now show that given the subsequent equilibrium strategies characterized in Step 1 and Step 2, the incumbent bank's equilibrium strategy in the primary loan market is to offer a pooling rate

$$R_N^* = \min\{R, R_N^E\} \tag{11}$$

regardless of the type of the borrower.

First, consider the interior solution $R_N^* = R_N^E < R$.[41] For the H-type borrower, the incumbent bank's expected profit when offering the loan rate $R_N^* = R_N^E$ is

$$\Pi_N(H) = [\pi + (1-\pi)(1-\rho)](R_N^E - r_N^I) > 0,$$

with $\Pi_N(H) > 0$ guaranteed by Assumption (2). Indeed, we have

$$c > \frac{1-\pi}{\pi} r_f \Rightarrow c > \frac{\alpha(1-\pi)\rho}{\pi + (1-\pi)\alpha(1-\rho)} r_f \Leftrightarrow \frac{c + r_f}{\alpha + (1-\alpha)\pi} > \frac{r_f}{\pi + (1-\pi)\alpha(1-\rho)}.$$

The incumbent bank has no profitable deviation. If the bank deviates by charging a loan rate $R^I(H) > R_N^E$, it will lose the loan competition and realizes a zero profit. If the bank deviates by charging a loan rate $R^I(H) < R_N^E$, it still wins the H-type borrower but earns an expected profit lower than $\Pi_N(H)$. For an L-type loan, the incumbent bank earns an expected profit equal to

$$\Pi_N(L) = \pi(R_N^E - r_N^I) > 0$$

by offering the equilibrium rate R_N^E. Again, the incumbent bank has no profitable deviation. If the bank deviates by charging a loan rate $R^I(L) > R_N^E$, it will lose the loan competition and realizes a zero profit. If the bank deviates by charging a loan rate $R^I(L) < R_N^E$, it still wins the L-type borrower but earns an expected profit lower than $\Pi_N(L)$.

[41] We assume the borrower sticks to the incumbent bank when there is a tie in competing loan rates.

Consider now the corner solution $R_N^* = R < R_N^E$. In this case, the incumbent bank charges a loan rate R independent of the borrower's type and earns an expected profit equal to

$$\Pi_N(H) = [\pi + (1-\pi)(1-\rho)](R - r_N^I)$$

and

$$\Pi_N(L) = \pi(R - r_N^I)$$

on the H- and L-type borrower, respectively. Both $\Pi_N(H)$ and $\Pi_N(L)$ are guaranteed to be positive by condition (3). Indeed, we have

$$R > \frac{r_f}{\pi} \Rightarrow R > \frac{r_f}{\pi + (1-\pi)\alpha(1-\rho)}.$$

Similar to the case of interior solution, one can verify the incumbent bank has no profitable deviation for both types.

Lastly, we show that, given its belief about the loan quality and the incumbent bank's strategy R_N^*, the entrant bank has no profitable deviation either. Consider first the interior case $R^E = R_N^E < R$. By offering a slightly higher loan rate $R_N^E + \varepsilon$, the entrant bank loses the loan market competition regardless of the borrower's type and realizes a zero profit. By offering a slightly lower loan rate $R^E = R_N^E - \varepsilon$, the entrant bank wins the borrower but would be better off in expectation by investing in the risk-free asset. Indeed, we have

$$[\alpha + (1-\alpha)\pi](R_N^E - \varepsilon) - (c + r_f) = -[\alpha + (1-\alpha)\pi]\varepsilon < 0.$$

Consider next the corner case where $R_N^E \geq R$. If the entrant bank offers a loan rate higher than R but below R_N^E, it will not be able to attract the borrower. The same occurs if the bank offers a loan rate that is even higher than R_N^E. If the entrant bank offers a rate that is lower than R, it wins the borrower regardless of its type but realizes a negative expected profit.

To summarize, we have established that the strategy profile and belief system described in Lemma 1 is indeed a pure-strategy PBE.

Part II: The uniqueness of the pure-strategy pooling PBE.

We consider all possible strategy profiles from the primary loan market competition and prove by contradiction that none of them can be part of a PBE. In other words, there exists no separating equilibrium or another pooling equilibrium other than the one described in

Lemma 1. Recall that R^E denote the loan rate offered by the entrant bank. In this section, we also denote $R^I(\theta)$ and $r^I(\theta)$ be the loan rate and deposit rate offered by the incumbent bank on a θ-type borrower, $\theta \in \{H, L\}$. We have the following four alternatives scenarios.

Scenario 1: Assume $R^E < \min\{R^I(H), R^I(L)\}$, so that the incumbent bank loses the loan market competition irrespective of the borrower's type. Suppose these loan rates had indeed been a part of a PBE. Then the incumbent bank offering a deposit rate to raise funding will be off the equilibrium path. We now show this cannot be a PBE, because the incumbent bank has profitable deviations for any off-equilibrium belief of depositors or asset buyers.

For the sake of the argument, consider the most pessimistic beliefs of depositors and asset buyers. That is, they believe that the incumbent bank must have financed an L-type loan if the bank had ever offered a deposit rate r^I or operated a loan sale. Given the belief, the asset buyers will offer a price equal to 0 in state B, and the depositors will accept a rate r_f/π to break even. Let us consider now the loan competition, given the incumbent bank's cost of funding. Note that when the entrant bank wins the loan competition irrespective of the borrower's type, R^E must satisfy

$$R^E \geq \frac{c + r_f}{\alpha + (1-\alpha)\pi}$$

so that the entrant bank's participation constraint is satisfied. The incumbent bank has profitable deviation. Consider the L-type borrower. By undercutting R^E, that is offering a rate $R^I(L) = R^E$, the incumbent bank's expected profit is positive

$$\pi\left(\frac{c + r_f}{\alpha + (1-\alpha)\pi} - \frac{r_f}{\pi}\right) > 0$$

because of Assumption (2). It is straightforward to see that profitable deviation also exists for an H-type borrower and for any more optimistic off-equilibrium beliefs of depositors and asset buyers. Therefore, $R^E < \min\{R^I(H), R^I(L)\}$ cannot be part of a PBE.

Scenario 2: Assume $R^I(H) \leq R^E < R^I(L)$, so that the incumbent bank only wins the H-type borrower in the primary loan market competition. Suppose these loan rates had indeed been a part of a PBE and that the incumbent bank offers a rate $r^I(H)$ to depositors. The depositors' on-equilibrium belief will be

$$Pr(H|r^I(H)) = 1.$$

Furthermore, the depositors expect asset buyers to Bayesian update the incumbent bank's loan quality according to the equilibrium strategy and to purchase the incumbent bank's asset on sale at a price $P^G = P^B = R^I(H) > r_f$. Therefore, it is sequentially rational for the depositors to provide financing for $r^I(H) = r_f$.

We now analyze the primary loan market. In such a separating equilibrium where the entrant bank only finances the L-type, the entrant bank's participation constraint requires

$$R^E \geq \frac{c + r_f}{\pi}.$$

Given its funding cost r_f, the incumbent bank can profitably deviates by increasing $R^I(H)$ until it reaches R^E for the H-type, given that

$$(R^E - r_f) - (R^I(H) - r_f) = R^E - R^I(H) \geq 0,$$

and by decreasing $R^I(L)$ until R^E for the L-type, given that

$$\pi \left(\frac{c + r_f}{\pi} - r_f \right) > 0.$$

Consequently, $R^I(H) \leq R^E < R^I(L)$ cannot be part of a PBE either.

Scenario 3: Assume $R^I(L) \leq R^E < R^I(H)$, so that the incumbent bank only wins the L-type borrower in loan market competition. Suppose these loan rates had indeed been a part of a PBE. Similar to Scenario 2, the depositors would hold on the equilibrium path the following belief

$$Pr(L|r^I(L)) = 1.$$

Depositors also expect the asset buyers to offer $P^G = R^I(L) > r_f$ and $P^B = 0$ for the incumbent bank's asset on sale in state G and B, respectively. Therefore, it is sequentially rational for the depositors to provide financing for $r^I(L) = r_f/\pi$.

In a separating equilibrium where the entrant bank now wins the H-type borrower, the bank's participation constraint requires

$$R^E \geq c + r_f.$$

Given its funding cost r_f/π, the incumbent bank can profitably deviate by increasing $R^I(L)$ until R^E for the L-type, given that

$$\pi \left(R^E - \frac{r_f}{\pi} \right) - \pi \left(R^I(L) - \frac{r_f}{\pi} \right) = \pi \left(R^E - R^I(L) \right) \geq 0,$$

and by decreasing $R^I(H)$ until R^E for the H-type, given that

$$c + r_f - \frac{r_f}{\pi} = c - \frac{1-\pi}{\pi} r_f > 0$$

because of Assumption (2). Therefore, we establish that $R^I(L) \leq R^E < R^I(H)$ cannot be part of a PBE.

Scenario 4: Assume $\max\{R^I(H), R^I(L)\} \leq R^E$, so that the incumbent bank still wins the loan market competition regardless of the borrower's type. But instead of a pooling equilibrium as described in Lemma 1, there is a separating PBE where either $R^I(H) \neq R^I(L)$, or $r^I(H) \neq r^I(L)$, or both. We prove by contradiction that these rates cannot be part of a PBE because the incumbent bank will have incentives to deviate. In particular, an incumbent bank with an L-type loan will mimic a bank with an H-type loan.

Suppose the incumbent bank offers separating deposit rates $r^I(H) \neq r^I(L)$ in equilibrium. The depositors' beliefs on the equilibrium path will be

$$Pr\left(H | r^I(H)\right) = 1 \quad \text{and} \quad Pr\left(L | r^I(L)\right) = 1,$$

and they will provide funding with deposit rates equal to $r^I(L) \geq r_f/\pi$ and $r^I(H) \geq r_f/[\pi + (1-\pi)(1-\rho)]$. As a result, the incumbent bank can profitably deviate by offering $r^I(L) = r^I(H) = r_f/[\pi + (1-\pi)(1-\rho)]$. That is, to secure a lower cost of funding, the incumbent bank would always claim having lend to an H-type borrower given the depositors' beliefs on the equilibrium path. Therefore, offering separating deposit rates must not be a part of PBE.

We now move one step backwards to analyze the primary loan market competition. Suppose separating loan rates $R^I(H) \neq R^I(L)$ had indeed been a part of a PBE. Note that the incumbent bank's funding cost for a loan will be r_N^I according to our previous discussion. Moreover, recall that the entrant bank's minimum profitable loan rate is R_N^E, so we must have $R^E \geq R_N^E$. Given such competing loan rate R^E and funding cost r_N^I, the incumbent bank has profitable deviations by increasing both $R^I(H)$ and $R^I(L)$ up to R^E if $\max\{R^I(H), R^I(L)\} < R^E$ or increasing the lowest until R^E if $\max\{R^I(H), R^I(L)\} = R^E$. To see this, note that

$$R^E - r_N^I \geq \min\{R, R_N^E\} - r_N^I > 0.$$

Consequently, offering the above separating loan rates can not be a sequential rational action for the incumbent bank.

In sum, we establish a unique pure-strategy pooling PBE as described in Lemma 1.

.2 Proof of Proposition 1

Depending on the value of R, we have four cases (note that the index given to each case represents the number of the interior solutions):

Case 0: $R \in [0, c+r_f)$ so that $R_S^*(H) = R_N^* = R_S^*(L) = R$;

Case 1: $R \in \left[c+r_f, \frac{c+r_f}{\alpha+(1-\alpha)\pi}\right)$ so that $R_S^*(H) = c+r_f$ and $R_N^* = R_S^*(L) = R$;

Case 2: $R \in \left[\frac{c+r_f}{\alpha+(1-\alpha)\pi}, \frac{c+r_f}{\pi}\right)$ so that $R_S^*(H) = c+r_f$, $R_N^* = \frac{c+r_f}{\alpha+(1-\alpha)\pi}$ and $R_S^*(L) = R$;

Case 3: $R \in \left[\frac{c+r_f}{\pi}, \infty\right)$ so that $R_S^*(H) = c+r_f$, $R_N^* = \frac{c+r_f}{\alpha+(1-\alpha)\pi}$ and $R_S^*(L) = \frac{c+r_f}{\pi}$.

For Case 0 and Case 3, in the text we have shown $V_S^{Type} \geq V_N$, independent of ρ, with the inequality being strict as long as $\rho > 0$. It remains to show that in Case 1 and Case 2 there exists a $\hat{\rho} \in (0,1)$ such that $V_S^{Type} > V_N$ for $\rho > \hat{\rho}$.

Consider Case 1. At $t = 0$, the incumbent bank's expected profit V_N can be written as

$$V_N = [\pi + (1-\pi)\alpha(1-\rho)]\, R - r_f,$$

which monotonically decreases in ρ. At the boundaries we have

$$\lim_{\rho \to 0} V_N = [\pi + (1-\pi)\alpha]\, R - r_f \quad \text{and} \quad \lim_{\rho \to 1} V_N = \pi R - r_f.$$

On the other hand, the expected profits V_S^{Type} can be written as

$$\begin{aligned} V_S^{Type} &= \alpha(c+r_f) + (1-\alpha)\pi R - r_f \\ &= \pi R + \alpha\left[(c+r_f) - \pi R\right] - r_f. \end{aligned}$$

The last expression is strictly smaller than $[\pi + (1-\pi)\alpha]\, R - r_f$ because under Case 1 we have $R > c+r_f$, and it is greater than $\pi R - r_f$ because $\pi R < (r_f + c)$ (a condition implied by the upper bound on R under Case 1). Since V_N is continuous in ρ, there must exist a $\hat{\rho}_1 \in (0,1)$ such that $V_S^{Type} > V_N$ for $\rho > \hat{\rho}_1$.

Consider Case 2. The expected payoff V_N becomes

$$V_N = \frac{\pi + (1-\pi)\alpha(1-\rho)}{\alpha + (1-\alpha)\pi}(c+r_f) - r_f,$$

which still monotonically decreases in ρ. At the boundaries we have

$$\lim_{\rho \to 0} V_N = c \quad \text{and} \quad \lim_{\rho \to 1} V_N = \frac{\pi(c+r_f)}{\alpha + (1-\alpha)\pi} - r_f.$$

On the other hand, the expression of V_S^{Type} remains the same as in Case 1, that is

$$V_S^{Type} = \alpha(c + r_f) + (1 - \alpha)\pi R - r_f < c$$

because $\pi R < (c + r_f)$ also in Case 2. Meanwhile, we know

$$V_S^{Type} > \alpha(c + r_f) + (1 - \alpha)\pi \frac{c + r_f}{\alpha + (1 - \alpha)\pi} - r_f = \left[\alpha + \frac{(1 - \alpha)\pi}{\alpha + (1 - \alpha)\pi}\right](c + r_f) - r_f,$$

where the inequality holds because under Case 2 we have $R > \frac{c+r_f}{\alpha+(1-\alpha)\pi}$. The last expression is greater than the lower bound of V_N because

$$\alpha + \frac{(1 - \alpha)\pi}{\alpha + (1 - \alpha)\pi} > \frac{\pi}{\alpha + (1 - \alpha)\pi}.$$

Therefore, since V_N is continuous in ρ, there must exist a $\hat{\rho}_2 \in (0, 1)$ such that $V_S^{Type} > V_N$ for $\rho > \hat{\rho}_2$. Letting $\hat{\rho} \leq \min\{\hat{\rho}_1, \hat{\rho}_2\}$, we have proven Proposition 1.

It is informative to see that information sharing increases the price of an H-type loan in state B. To see so, notice that $P_S^B(H) = c + r_f$ and $P_N^B = \frac{\alpha\rho}{(1-\alpha)+\alpha\rho} R_N^*$. For $R_N^* = \min\left\{R_N^E, R\right\}$ and

$$\frac{\alpha\rho}{(1 - \alpha) + \alpha\rho} R_N^E = \frac{\alpha\rho}{(1 - \alpha) + \alpha\rho} \cdot \frac{c + r_f}{\alpha + (1 - \alpha)\pi} < c + r_f,$$

we know $P_N^B < P_S^B(H)$ must be true. When $r_f \in \left(P_N^B, P_S^B(H)\right)$, by sharing information on the borrower's type, the incumbent bank will be saved from runs in state B.

.3 Proof of Lemma 2

When the incumbent bank shares the borrower's credit history and lends to a borrower with no previous default ($\overline{D}$), the game also features incomplete information because such a borrower still can be either an H- or L-type. Therefore, we apply the solution concept of PBE. Different from the previous case where they hold the prior about the borrower's type, the outsiders, i.e., the entrant bank, depositors and asset buyers, now can update their belief at the initial node of $g_S(\overline{D})$ according to the shared credit history $\overline{D}$. At that point of time, their belief about the borrower's type, $\mu(\overline{D})$, consists of the following conditional probabilities:

$$Pr(H|\overline{D}) = \frac{Pr(H, \overline{D})}{Pr(\overline{D})} = \frac{\alpha}{\alpha + (1 - \alpha)\delta} > \alpha, \tag{12}$$

$$Pr(L|\overline{D}) = \frac{Pr(L, \overline{D})}{Pr(\overline{D})} = \frac{(1 - \alpha)\delta}{\alpha + (1 - \alpha)\delta} < 1 - \alpha. \tag{13}$$

In this section, we denote by $R^I\left(\theta, \overline{D}\right)$ and $r^I\left(\theta, \overline{D}\right)$ the loan rate and deposit rate offered by the incumbent bank on a θ-type borrower with a $\overline{D}$-history, $\theta \in \{H, L\}$. Let $R^E(\overline{D})$ denote the loan rate offered by the entrant bank. We show that the unique pure-strategy equilibrium is, again, a pooling equilibrium, as the incumbent bank offers a unified loan rate $R^I_S(\overline{D}) \in [r_f, R]$ and a unified deposit rate $r^I_S(\overline{D}) \in [r_f, r_f/\pi]$, independent of the type of borrower it finances.

Definition 2. *A pure strategy pooling PBE of the game $g_S(\overline{D})$ is characterized as the following. (i) An equilibrium strategy profile. Based on its knowledge of the borrower's type, the incumbent bank at $t = 2$ sets a loan rate $R^I_S(\overline{D})$ for the borrower with no default history and offers a take-it-or-leave-it rate $r^I_S(\overline{D})$ to depositors. When having financed the borrower, the incumbent bank decides at $t = 3$ whether to sell the loan, according to the loan quality, the state s, and its own liquidity position. The entrant bank offers a competing loan rate $R^E(\overline{D})$ to the borrower with no previous default. Depositors choose to provide funding or not, based on the offered deposit rate and the borrower's credit history of no default. Asset buyers bid $P^G_S(\overline{D})$ in state G and $P^B_S(\overline{D})$ in state B to purchase the loan if it is on sale.*
(ii) A system of beliefs. The entrant bank holds the belief $\mu(\overline{D})$ about loan quality. Seeing that the borrower has no previous default, the depositors hold a posterior belief $\mu\left(\overline{D}, r^I_S(\overline{D})\right)$ about loan quality when receiving incumbent's deposit rate $r^I_S(\overline{D})$. Asset buyers Bayesian update their beliefs $\mu\left(\overline{D}, s\right)$ for the borrower who has no default history, according to both the aggregate state and observation of asset sale.
(iii) The strategy profile in (i) is sequential rational given the beliefs in (ii).

The proof again consists of two parts. Part I establishes the equilibrium described in Lemma 2. Part II establishes the uniqueness of the PBE.

Part I: The existence of a pure-strategy pooling PBE. To derive the equilibrium described in Lemma 2, we solve the game backwards.

Step 1. We start by analyzing the secondary loan market in the state G and B respectively. Given the shared borrower credit history $\overline{D}$ and the incumbent bank's equilibrium strategy described in Lemma 2, the asset buyers' beliefs in the state G are characterized by equation (12) and (13). Since both an H- and L-type borrower will repay $R^*_S(\overline{D})$ in state G, asset buyers' competitive bidding leads to the following break-even condition

$$Pr(H|\overline{D})\left[R_S^*(\overline{D})-P_S^G(\overline{D})\right]+Pr(L|\overline{D})\left[R_S^*(\overline{D})-P_S^G(\overline{D})\right]=0,$$

which implies

$$P_S^G(\overline{D})=R_S^*(\overline{D}). \tag{14}$$

In state B, the incumbent bank always sells its L-type loan and sells an H-type loan only when facing a run, and asset buyers Bayesian update their belief accordingly

$$Pr(H|\text{loan sale},\overline{D})=\frac{Pr(run)Pr(H|\overline{D})}{Pr(run)Pr(H|\overline{D})+Pr(L|\overline{D})}=\frac{Pr(H|\overline{D})\rho}{Pr(H|\overline{D})\rho+Pr(L|\overline{D})}.$$

Since only the H-type borrower will repay $R_S^*(\overline{D})$ in state B, the asset buyers' competitive bidding leads to the following break-even condition

$$\frac{Pr(H|\overline{D})\rho}{Pr(H|\overline{D})\rho+Pr(L|\overline{D})}\left[R_S^*(\overline{D})-P_S^B(\overline{D})\right]+\frac{Pr(L|\overline{D})}{Pr(H|\overline{D})\rho+Pr(L|\overline{D})}\left[0-P_S^B(\overline{D})\right]=0.$$

Substituting in the expression of $Pr(H|\overline{D})$ and that of $Pr(L|\overline{D})$, we obtain

$$P_S^B(\overline{D})=\frac{\alpha\rho}{(1-\alpha)\delta+\alpha\rho}R_S^*(\overline{D}). \tag{15}$$

Similar to the proof of Lemma 1, one can verify that asset buyers' beliefs are consistent with the incumbent bank's equilibrium strategy, and the incumbent bank's loan sale decision is sequentially rational given the asset buyers' equilibrium bidding.

Step 2. We move to the stage when the incumbent bank raises its funding, and analyze the signaling game between the incumbent bank and its depositors.

We again start with depositors' belief and strategy on the equilibrium path described by Lemma 2. As the incumbent bank offers a pooling deposit rate $r_S^I(\overline{D})$, the depositors' belief about the bank's loan quality remains the same as $\mu(\overline{D})$. That is,

$$Pr\left(H|r_S^I(\overline{D}),\overline{D}\right)=Pr(H|\overline{D})\quad\text{and}\quad Pr\left(L|r_S^I(\overline{D}),\overline{D}\right)=Pr(L|\overline{D}).$$

Furthermore, under the condition

$$P_S^B(\overline{D})=\frac{\alpha\rho}{(1-\alpha)\delta+\alpha\rho}R_S^*(\overline{D})>r_f,$$

the incumbent bank can raise enough liquidity from the loan sale when a run happens in the state B. The depositors therefore anticipate the bank to fully repay its debt in both states. They are willing to accept the risk-free rate

$$r_S^I(\overline{D}) = r_f. \tag{16}$$

Recall that we assume depositors' off-equilibrium belief about the loan quality to be worse than the prior. Therefore, the belief is worse than $\mu(\overline{D})$. To break even given such beliefs, the depositors must demand a rate that is higher than $r_S^I(\overline{D}) = r_f$.

Given the depositors' break-even rates, we show that it is indeed sequentially rational for the incumbent bank to offer the pooling equilibrium deposit rate $r_S^I(\overline{D}) = r_f$.

When the incumbent bank finances an H-type loan with deposit rate $r_S^I(\overline{D}) = r_f$, it makes an expected profit

$$\begin{aligned} \Pi_S(H,\overline{D}) &= \pi\left(R_S^*(\overline{D}) - r_f\right) + (1-\pi)\left[\rho\left(P_B^S(\overline{D}) - r_f\right) + (1-\rho)\left(R_S^*(\overline{D}) - r_f\right)\right] \\ &= \left[\pi + (1-\pi)(1-\rho)\right]\left(R_S^*(\overline{D}) - r_f\right) + (1-\pi)\rho\left(P_B^S(\overline{D}) - r_f\right) > 0. \end{aligned} \tag{17}$$

If the bank deviates by offering a deposit rate $r^I(\overline{D}) \neq r_S^I(\overline{D})$ to the depositors, its expected profit is either

$$\hat{\Pi}_S(H,\overline{D}) = 0 \quad \text{if } \; r^I(\overline{D}) < r_f,$$

or

$$\begin{aligned} \hat{\Pi}_S(H,\overline{D}) =& \left[\pi + (1-\pi)(1-\rho)\right]\left(R_S^*(\overline{D}) - r^I(\overline{D})\right) + (1-\pi)\rho\left(P_B^S(\overline{D}) - r^I(\overline{D})\right) \\ &< \Pi_S(H,\overline{D}) \quad \text{if } \; r^I(\overline{D}) > \overline{D}) = r_f. \end{aligned}$$

Therefore, the bank has no incentive to deviate from $r^I(\overline{D})$ when holding an H-type loan.

When the incumbent bank finances an L-type loan with the deposit rate $r_S^I(\overline{D}) = r_f$, it earns an expected profit

$$\Pi_S(L,\overline{D}) = \pi\left(R_S^*(\overline{D}) - r_f\right) + (1-\pi)\left(P_B^S(\overline{D}) - r_f\right) > 0. \tag{18}$$

By deviating and offering a rate $r^I(\overline{D}) \neq r_f$, the bank earns either

$$\hat{\Pi}_S(L,\overline{D}) = 0 \quad \text{if } \; r^I(\overline{D}) < r_f,$$

or

$$\hat{\Pi}_S(L,\overline{D}) = \pi\left(R_S^*(\overline{D}) - r^I(\overline{D})\right) + (1-\pi)\left(P_B^S(\overline{D}) - r^I(\overline{D})\right) < \Pi_S(L,\overline{D}) \quad \text{if } \; r^I(\overline{D}) > r_f.$$

The incumbent bank has no profitable deviation when holding an L-type loan either.

Step 3. We now analyze the primary loan market competition between the incumbent and entrant bank.

Given the entrant bank's belief $\mu(\overline{D})$, the minimum loan rate that satisfies the participation constraint of the entrant bank is

$$R_S^E(\overline{D}) = \frac{\alpha + (1-\alpha)\delta}{\alpha + (1-\alpha)\delta\pi}(c + r_f). \tag{19}$$

Otherwise, the entrant bank will be better off holding the risk-free asset. That is,

$$\left[Pr(H|\overline{D}) + Pr(L|\overline{D})\pi\right] R^E(\overline{D}) - c = \frac{\alpha + (1-\alpha)\delta\pi}{\alpha + (1-\alpha)\delta} R^E(\overline{D}) - c < r_f \quad \forall R^E(\overline{D}) < R_S^E(\overline{D}).$$

We now show that given the subsequent equilibrium strategies characterized in Step 1 and Step 2, the incumbent bank's equilibrium strategy in the primary loan market is to offer a pooling rate

$$R_S^*(\overline{D}) = \min\{R, R_S^E(\overline{D})\}, \tag{20}$$

regardless of the type of the borrower.

First, consider the interior solution $R_S^*(\overline{D}) = R_S^E(\overline{D}) < R$. Our presumption $P_S^B(\overline{D}) > r_f$ ensures a positive expected profit for the incumbent bank, regardless the type of the borrower it finances.[42]

Suppose the incumbent bank deviates by charging a loan rate $R^I(H,\overline{D}) > R_S^E(\overline{D})$. The bank will lose the loan competition for the H-type borrower and realizes a zero-profit

$$\hat{\Pi}_S(H,\overline{D}) = 0.$$

If the bank deviates by charging a loan rate $R^I(H,\overline{D}) < R_S^E(\overline{D})$, it will still win the H-type borrower but will reduce its expected profit compared to the equilibrium level.[43] That is,

$$\hat{\Pi}_S(H,\overline{D}) = [\pi + (1-\pi)(1-\rho)]\left(R^I(H,\overline{D}) - r_f\right) + (1-\pi)\rho\left(P_B^S(\overline{D}) - r_f\right) < \Pi_S(H,\overline{D}).$$

[42] Note that the incumbent bank's equilibrium profits are positive and expressed in (17) and (18).

[43] Note that in the expression of $\hat{\Pi}_S(H,\overline{D})$, the only deviation from $\Pi_S(H,\overline{D})$ is the loan rate $R^I(H,\overline{D})$. The asset price $P_S^B(\overline{D})$ is not affected as we assume that the asset buyers can not observe the loan rate charged by the incumbent bank. Thus, the buyers still believe the loan rate is the equilibrium rate $R_S^*(\overline{D}) = R_S^E(\overline{D})$.

For an L-type borrower, the incumbent bank's expected profit by deviating is either

$$\hat{\Pi}_S(L,\overline{D}) = 0 < \Pi_S(L,\overline{D}) \quad \text{if} \quad R^I(L,\overline{D}) > R_S^E(\overline{D}),$$

or

$$\hat{\Pi}_S(L,\overline{D}) = \pi \left(R^I(L,\overline{D}) - r_f\right) + (1-\pi)\left(P_B^S(\overline{D}) - r_f\right) < \Pi_S(L,\overline{D}) \quad \text{if} \quad R^I(L,\overline{D}) < R_S^E(\overline{D}).$$

Therefore, the incumbent has no profitable deviation for either the H- or L-type borrower.

For the corner solution $R_S^*(\overline{D}) = R > R_S^E(\overline{D})$, the incumbent bank charges the entire cash flow of the borrower's project R. Under condition $P_S^B(\overline{D}) > r_f$, the incumbent bank's expected profit for both the H- and the L-type loan is positive. Following the same argument, one can verify that the incumbent bank has no profitable deviation in the corner solution case.

Lastly, one can follow the same argument in the proof of Lemma 1 to show that the entrant bank has no profitable deviation by offering any loan rate other than $R_S^E(\overline{D})$ given its belief $\mu(\overline{D})$.

To summarize, we establish the strategy profile and belief system described in Lemma 2 is indeed a pure-strategy PBE.

Part II: The uniqueness of the pure-strategy pooling PBE.

We again consider all possible alternative pure-strategy profiles in the primary loan market competition and prove by contradiction that none of them can be part of a PBE.

Scenario 1: Assume $R^E(\overline{D}) < \min\{R^I(H,\overline{D}), R^I(L,\overline{D})\}$, so that the incumbent bank loses the loan market competition for the borrower with $\overline{D}$ credit history irrespective of the borrower's type. Suppose these loan rates had indeed been a part of a PBE. Then the incumbent bank offering a deposit rate to raise funding will be off the equilibrium path. We now show this cannot be a PBE, because the incumbent bank has profitable deviations for any off-equilibrium belief of depositors or asset buyers.

Consider the most pessimistic beliefs of depositors and asset buyers. That is, they believe that the incumbent bank must have financed an L-type loan if the bank had ever offered a deposit rate r^I or conducted a loan sale. Given the belief, the asset buyers will offer a price equal to 0 in the state B, and the depositors will accept a rate r_f/π to break even. Consider now the primary loan market competition, given the incumbent bank's cost of funding. Note that when the entrant bank wins the loan competition irrespective of the borrower's type, $R^E(\overline{D})$ must satisfy

$$R^E(\overline{D}) \geq \frac{\alpha + (1-\alpha)\delta}{\alpha + (1-\alpha)\delta\pi}(c + r_f) = R^E_S(\overline{D})$$

so that the entrant bank's participation constraint is satisfied. The incumbent bank has profitable deviation for the L-type borrower. By undercutting the entrant bank and offering a loan rate equal to $R^E_S(\overline{D})$, the incumbent bank makes a positive expected profit

$$\pi\left(\frac{\alpha + (1-\alpha)\delta}{\alpha + (1-\alpha)\delta\pi}(c + r_f) - \frac{r_f}{\pi}\right) > 0,$$

because the inequality is equivalent to $c > \frac{\alpha(1-\pi)}{\alpha\pi + (1-\alpha)\delta\pi} r_f$ and is implied by Assumption (2). Also, it is possible to show that a profitable deviation exists for an H-type and for any more optimistic beliefs off the equilibrium path. Therefore, $R^E(\overline{D}) < \min\{R^I(H, \overline{D}), R^I(L, \overline{D})\}$ cannot be part of a PBE.

Scenario 2: Assume $R^I(H, \overline{D}) \leq R^E(\overline{D}) < R^I(L, \overline{D})$, so that the incumbent bank only wins the H-type borrower in loan market competition. Suppose these loan rates had indeed been a part of a PBE and that the incumbent bank offers a rate $r^I(H, \overline{D})$ to depositors. The depositors' on-equilibrium belief will be

$$Pr(H|r^I(H, \overline{D})) = 1.$$

Furthermore, the depositors expect asset buyers to Bayesian update the incumbent bank's loan quality according to the equilibrium strategy and to purchase the incumbent bank's asset on sale at a price $P^G = P^B = R^I(H, \overline{D}) > r_f$. Therefore, it is sequentially rational for the depositors to provide financing at the risk-free rate: $r^I(H, \overline{D}) = r_f$.

We now analyze the primary loan market. In such a separating equilibrium where the entrant bank only finances the L-type, the entrant bank's participation constraint requires

$$R^E(\overline{D}) \geq \frac{c + r_f}{\pi}.$$

Given its funding cost r_f, the incumbent bank can profitably deviate by increasing $R^I(H, \overline{D})$ to $R^E(\overline{D})$ for the H-type, given that

$$\left(R^E(\overline{D}) - r_f\right) - \left(R^I(H, \overline{D}) - r_f\right) = R^E(\overline{D}) - R^I(H, \overline{D}) \geq 0.$$

The incumbent bank can also profitably deviate by decreasing $R^I(L, \overline{D})$ until it reaches $R^E(\overline{D})$ for the L-type, given that

$$\pi\left(\frac{c + r_f}{\pi} - r_f\right) > 0.$$

Consequently, $R^I(H,\overline{D}) \leq R^E(\overline{D}) < R^I(L,\overline{D})$ cannot be part of a PBE either.

Scenario 3: Assume $R^I(L,\overline{D}) \leq R^E(\overline{D}) < R^I(H,\overline{D})$, so that the incumbent bank only wins the L-type borrower in loan market competition. Suppose these loan rates had indeed been a part of a PBE. The depositors would hold an on-equilibrium belief

$$Pr(L|r^I(L,\overline{D})) = 1,$$

and expect asset buyers to offer $P^G = R^I(L,\overline{D}) > r_f$ and $P^B = 0$ for the incumbent bank's asset on sale in state G and B, respectively. Therefore, it is sequentially rational for the depositors to provide financing for $r^I(L,\overline{D}) = r_f/\pi$.

In such a separating equilibrium where the entrant bank now wins only the H-type borrower, its participation constraint requires

$$R^E(\overline{D}) \geq c + r_f.$$

Given its funding cost r_f/π, the incumbent bank can profitably deviates by increasing $R^I(L,\overline{D})$ to $R^E(\overline{D})$ for the L-type, given that

$$\pi\left(R^E(\overline{D}) - \frac{r_f}{\pi}\right) > \pi\left(R^I(L,\overline{D}) - \frac{r_f}{\pi}\right) = \pi\left(R^E(\overline{D}) - R^I(L,\overline{D})\right) \geq 0.$$

The incumbent bank can also profitably deviates by decreasing $R^I(H,\overline{D})$ to $R^E(\overline{D})$ for the H-type, given that

$$R^E(\overline{D}) - \frac{r_f}{\pi} \geq c + r_f - \frac{r_f}{\pi} > 0.$$

Consequently, $R^I(L,\overline{D}) \leq R^E(\overline{D}) < R^I(H,\overline{D})$ cannot be part of a PBE.

Scenario 4: Assume $\max\{R^I(H,\overline{D}), R^I(L,\overline{D})\} \leq R^E(\overline{D})$, so that the incumbent bank still wins the loan market competition for the borrower with a credit history of $\overline{D}$, regardless of the borrower's type. However, instead of a pooling equilibrium as described in Lemma 2, a separating PBE exists where either $R^I(H,\overline{D}) \neq R^I(L,\overline{D})$, or $r^I(H,\overline{D}) \neq r^I(L,\overline{D})$, or both.

Suppose the incumbent bank offers separating deposit rates $r^I(H,\overline{D}) \neq r^I(L,\overline{D})$ in the equilibrium. The depositors' belief on the equilibrium path would be

$$Pr(H|r^I(H,\overline{D})) = 1 \quad \text{and} \quad Pr(L|r^I(L,\overline{D})) = 1.$$

In this case, depositors are willing to provide funding with deposit rates $r^I(L,\overline{D}) \geq r_f$ and $r^I(H,\overline{D}) \geq r_f$ when they anticipate the loan price to be $P_S^B(\overline{D}) \geq r_f$ in the state B.

Otherwise, they provide funding for deposit rates $r^I(L, \overline{D}) \geq r_f/\pi$ and $r^I(H, \overline{D}) \geq r_f/\pi$ when they anticipate the loan price to be $P_S^B(\overline{D}) < r_f$ in the state B. As a result, to secure a lower cost of funding, the incumbent bank would always claim having lend to the type of borrower with lower deposit rate (not necessarily the H-type in this case). Therefore, offering separating deposit rates must not be a part of any PBE.

We now move to the primary loan market. Suppose separating loan rates $R^I(H, \overline{D}) \neq R^I(L, \overline{D})$ had indeed been a part of a PBE. Then there must be a profitable deviation for the incumbent bank to increase the lower loan rate until the the two are rates equal.

To summarize, we establish the strategy profile and beliefs in Lemma 2 as a unique pure-strategy PBE for the game $g_S(\overline{D})$.

.4 Proof of Lemma 3

When the incumbent bank shares the borrower's credit history with default D, the game features complete information. Indeed, the H-type borrower is assumed to not default therefore the outsiders Bayesian update their beliefs as $Pr(L|D) = 1$. Then we apply the solution concept of SPE and define a pure-strategy SPE of the game $g_S(D)$ as follows.[44]

Definition 3: *In a pure-strategy SPE of the game $g_S(D)$, the incumbent bank at $t = 2$ sets a loan rate $R_S^I(D)$ and offers a take-it-or-leave-it deposit rate $r_S^I(D)$. When having financed the borrower, the incumbent bank decides at $t = 3$ whether to sell the loan, according to the aggregate state s and its own liquidity position. Knowing that the borrower is an L-type, The entrant bank sets a loan rate $R_S^E(D)$. Also knowing the loan is of an L-type, depositors decide to provide funding or not for the deposit rate offered. When the L-type loan is on sale, the asset buyers bid $P_S^G(D)$ in state G and $P_S^B(D)$ in state B.*

We solve the game backwards in three steps.

Step 1. We first derive the secondary-market price for a D-history loan in State G and B respectively. In State B, asset buyers understand that the loan will produce a 0 return with certainty, which leads to the unique price

$$P_S^B(D) = 0.$$

In State G, asset buyers' competitive bidding implies the unique price

$$P_S^G(D) = R_S^*(D)$$

[44] For notation, we express the conditionality in terms of credit history D instead of the true type L.

because an L-type loan does not default in state G.

Step 2. We move to the stage when the incumbent bank raises its funding. Different from the previous cases in Lemma 1 and 2, now the depositors are perfectly informed about the loan quality. Their *unique* break-even rate is

$$r_S^I(D) = \frac{r_f}{\pi},$$

since the bank defaults with certainty in state B. So the optimal strategy for the incumbent bank is to offer $r_S^I(D)$ to make the depositors break even.

Step 3. We now analyze the primary loan market competition between the incumbent and entrant banks.

Given the entrant bank's posterior belief $Pr(L|D) = 1$, the minimum loan rate that satisfies the entrant bank's participation constraint is

$$R_S^E(D) = \frac{c + r_f}{\pi}.$$

By the standard argument of the price competition, the unique equilibrium in the primary loan market involves the incumbent bank offering

$$R_S^*(D) = \min\left\{R,\, R_S^E(D)\right\}$$

and the entrant bank offering rate $R_S^E(D)$. Given the incumbent bank's funding cost r_f/π, it makes a positive expected profit

$$\Pi_S(D) = \min\left\{R, \frac{c + r_f}{\pi}\right\} - \frac{r_f}{\pi} > 0$$

as guaranteed by condition (3).

To summarize, we establish the strategy profile in Lemma 3 as a unique pure-strategy SPE for the game $g_S(D)$.

.5 Proof of Lemma 4

The equilibrium prices of the loan on sale in the secondary market are

$$P_N^B = \frac{\alpha\rho}{(1-\alpha) + \alpha\rho} R_N^*$$

and

$$P_S^B(\overline{D}) = \frac{\alpha\rho}{(1-\alpha)\delta + \alpha\rho} R_S^*(\overline{D}),$$

where R_N^* and $R_S^*(\overline{D})$ are the equilibrium loan rates without and with information sharing and information, respectively. The expressions for equilibrium loan rates depend on the loan's payoff R and the specific case under analysis.

Consider Case 0. The return R is so low that the entrant bank does not compete for any loan even if the incumbent bank shared a $\overline{D}$ credit history of the borrower. The incumbent bank extracts the entire payoff of the loan irrespective of the information sharing regime, that is $R_S^*(\overline{D}) = R_N^* = R$. Information sharing solely brings in the benefit from boosting liquidity of the loan on sale with a $\overline{D}$-history. Consequently, $P_S^B(\overline{D}) > P_N^B$.

Consider Case 2 (for the easy of exposition it is convenient to prove this case first). The value of R is sufficiently high that the entrant bank competes both under information sharing (when the borrower has no previous default) and under no information sharing. The equilibrium loan rates are therefore

$$R_N^* = R_N^E = \frac{c + r_f}{\alpha + (1-\alpha)\pi} > \frac{\alpha + (1-\alpha)\delta}{\alpha + (1-\alpha)\pi\delta}(c + r_f) = R_S^E(\overline{D}) = R_S^*(\overline{D}).$$

We want to show that

$$P_N^B = \frac{\alpha\rho}{(1-\alpha) + \alpha\rho}\frac{c + r_f}{\alpha + (1-\alpha)\pi} < \frac{\alpha\rho}{(1-\alpha)\delta + \alpha\rho}\frac{\alpha + (1-\alpha)\delta}{\alpha + (1-\alpha)\pi\delta}(c + r_f) = P_S^B(\overline{D}),$$

which can be rewritten as

$$\frac{\alpha\rho + (1-\alpha)\delta}{\alpha\rho + (1-\alpha)}\frac{\alpha + (1-\alpha)\pi\delta}{[\alpha + (1-\alpha)\delta][\alpha + (1-\alpha)\pi]} < 1. \tag{21}$$

To show that the inequality (21) holds, notice that the ratio $\frac{(1-\alpha)\delta + \alpha\rho}{(1-\alpha) + \alpha\rho}$ increases in ρ. So its maximum value is reached when $\rho = 1$ and equals $\alpha + (1-\alpha)\delta$. Therefore, an upper bound of the LHS of (21) can be written as

$$[\alpha + (1-\alpha)\delta]\frac{\alpha + (1-\alpha)\pi\delta}{[\alpha + (1-\alpha)\delta][\alpha + (1-\alpha)\pi]} = \frac{\alpha + (1-\alpha)\pi\delta}{\alpha + (1-\alpha)\pi}.$$

The expression is smaller than 1 for $\forall\delta \in (0,1)$, so that $P_S^B(\overline{D}) > P_N^B$.

Consider Case 1. The entrant bank only competes for the borrower with a revealed $\overline{D}$-history, such that $R_S^*(\overline{D}) = R_S^E(\overline{D})$ remains true in equilibrium. With no information sharing, however, the incumbent bank can charge to the borrower the entire project return by setting $R_N^* = R$, because $R < R_N^E$ and the entrant bank does not bid for the borrower.

Thus, the loss in information rent from primary loan market competition is smaller than that in Case 2, and $P_S^B(\overline{D}) > P_N^B$ also holds in Case 1.

Consider Case 3. The loan payoff R is sufficiently high that the entrant bank competes even when the loan is granted to a borrower with a default credit history D. The relevant equilibrium loan rates R_N^* and $R_S^*(\overline{D})$ are the same as in Case 2 and the same analysis applies. Therefore, $P_S^B(\overline{D}) > P_N^B$ also holds in Case 3.

.6 Proof of Lemma 5

Given $P_S^B(\overline{D}) > P_N^B$, by continuity, when r_f is located between the two prices, the incumbent bank survives a run in state B when the loan has a revealed credit history $\overline{D}$ and it fails when sharing no information. We now characterize the regions $\mathbb{F}_j$, with $j = \{0, 1, 2, 3\}$, where the inequality $P_S^B(\overline{D}) > r_f > P_N^B$ holds.

In Case 0, we have $R_N^* = R_S^*(\overline{D}) = R$, so that the inequality can be written as

$$\frac{\alpha\rho}{(1-\alpha)\delta + \alpha\rho} R > r_f > \frac{\alpha\rho}{(1-\alpha) + \alpha\rho} R$$

which is satisfied for $\forall \delta \in (0, 1)$. The nonempty set $\mathbb{F}_0$ is characterized by the boundaries $\underline{R}$ and $\overline{R}$ as defined as follows

$$R > \frac{\alpha\rho + (1-\alpha)\delta}{\alpha\rho} r_f \equiv \underline{R} \quad \text{and} \quad R < \frac{\alpha\rho + (1-\alpha)}{\alpha\rho} r_f \equiv \overline{R}.$$

The set Ψ_0 is non-empty, since $\mathbb{R}_0$ is delineated by $c + r_f$ and $R_S^E(\overline{D})$, both going through the origin.

In Case 1, we have $R_S^*(\overline{D}) = R_S^E(\overline{D})$ and $R_N^* = R$, so that the inequality becomes

$$\frac{\alpha\rho}{(1-\alpha)\delta + \alpha\rho} \frac{\alpha + (1-\alpha)\delta}{\alpha + (1-\alpha)\delta\pi} (c + r_f) > r_f > \frac{\alpha\rho}{(1-\alpha) + \alpha\rho} R.$$

The nonempty set $\mathbb{F}_1$ is characterized by boundaries $\underline{R}$ and $\underline{c} < c + r_f$, with $\underline{c}$ defined as follows

$$c > \left[\frac{\alpha\rho + (1-\alpha)\delta}{\alpha\rho} \frac{\alpha + (1-\alpha)\delta\pi}{\alpha + (1-\alpha)\delta} - 1 \right] r_f \equiv \underline{c}. \tag{22}$$

The set Ψ_1 is non-empty, since $\mathbb{R}_1$ is delineated by $R_S^E(\overline{D})$ and R_N^E, both going through the origin.

In Case 2, we have $R_S^*(\overline{D}) = R_S^E(\overline{D})$ and $R_N^* = R_N^E$ so that the inequality becomes

$$\frac{\alpha\rho}{(1-\alpha)\delta + \alpha\rho} \frac{\alpha + (1-\alpha)\delta}{\alpha + (1-\alpha)\delta\pi} (c + r_f) > r_f > \frac{\alpha\rho}{(1-\alpha) + \alpha\rho} \frac{c + r_f}{\alpha + (1-\alpha)\pi}.$$

The nonempty set $\mathbb{F}_2$ is characterized by the boundaries $\underline{c}$ and $\overline{c} > c + r_f$, with $\overline{c}$ defined as

$$c < \left[(\alpha + (1-\alpha)\pi) \frac{\alpha\rho + (1-\alpha)}{\alpha\rho} - 1 \right] r_f \equiv \overline{c}. \tag{23}$$

The set Ψ_2 is non-empty, since $\mathbb{R}_2$ is delineated by R_N^E and $R_S^E(D)$, both going through the origin.

In Case 3, the relevant equilibrium loan rates $R_S^*(\overline{D})$ and R_N^* are the same as in Case 2. Therefore we obtain the same cutoff values as in (22) and (23), so that the non-emptiness of $\mathbb{F}_3$ and Ψ_3 follow.

We now show that when

$$\pi > \hat{\pi} \equiv \frac{-\alpha + \sqrt{\alpha^2 + 4\alpha(1-\alpha)\delta}}{2(1-\alpha)\delta} \in (0,1) \tag{24}$$

the inequalities $(1-\pi)r_f/\pi < \underline{c}$ and $r_f/\pi < \underline{R}$ hold. That is, our parametric assumptions (1) and (2) are non-binding. To see so, notice that the former inequality implies the latter. Further notice that $\underline{c}$ decreases in ρ. A sufficient condition for $(1-\pi)r_f/\pi < \underline{c}$ is therefore

$$\frac{1-\pi}{\pi} r_f < \left[\frac{\alpha + (1-\alpha)\delta\pi}{\alpha} - 1 \right] r_f,$$

which gives a quadratic equation for the critical $\hat{\pi}$ as given in (24).

.7 Proof of Proposition 2

We determine the expected profits V_i in $t = 0$ of the incumbent bank in each information sharing regime $i = \{N, S\}$. In information sharing regime N, we have the expected profit

$$V_N = [\Pr(G) + \Pr(B)\Pr(H)\Pr(\text{no run})](R_N^* - r_N^I) = [\pi + (1-\pi)\alpha(1-\rho)]R_N^* - r_f.$$

In state G, the incumbent bank will survive, irrespective of the type of its borrower. However, in state B, the bank–even when holding an H-type loan–will survive only if no run happens. In the no information sharing regime N, the equilibrium deposit rate r_N^I is risky and given in Lemma 1. To facilitate comparison, we re-write V_N as follows

$$V_N = [\alpha + (1-\alpha)\delta\pi]R_N^* + (1-\alpha)(1-\delta)\pi R_N^* - \alpha(1-\pi)\rho R_N^* - r_f.$$

When the incumbent bank participates in the information sharing regime, its expected profits can be formulated as follows

$$V_S = \Pr(\overline{D}) \cdot \left[\Pr(H|\overline{D})\Pi_S(H,\overline{D}) + \Pr(L|\overline{D})\Pi_S(L,\overline{D})\right] + \Pr(D) \cdot \Pi_S(D). \qquad (25)$$

The expressions $\Pi_S(H,\overline{D})$ and $\Pi_S(L,\overline{D})$ are the expected profits of financing an H-type and an L-type borrower, respectively, when they generate a no default credit history $\overline{D}$. While $\Pi_S(D)$ represents the expected profit of financing an L-type borrower with a default credit history D. Notice that when a loan has a credit history $\overline{D}$, with posterior probability $\Pr(H|\overline{D})$ it is an H-type loan. Moreover, $\Pr(D) = (1-\alpha)(1-\delta)$ and $\Pr(\overline{D}) = 1 - \Pr(D) = \alpha + (1-\alpha)\delta$.

The expected profit of financing an H-type borrower with credit history $\overline{D}$ is

$$\Pi_S(H,\overline{D}) = [\Pr(G) + \Pr(B)\Pr(\text{no run})]R^*_S(\overline{D}) + \Pr(B)\Pr(\text{run})P^B_S(\overline{D}) - r_f.$$

Notice $r^I_S(\overline{D}) = r_f$, given that we focus on the case in which information sharing saves the incumbent bank from illiquidity. Moreover, the incumbent bank holds an H-type loan to maturity if no bank run occurs, because $P^B_S(\overline{D}) < R^*_S(\overline{D})$ (see Lemma 2). Similarly, the expected profit of financing an L-type borrower with credit history $\overline{D}$ is given by

$$\Pi_S(L,\overline{D}) = \Pr(G)R^*_S(\overline{D}) + \Pr(B)P^B_S(\overline{D}) - r_f.$$

When the incumbent bank holds an L-type loan, it will sell it on the secondary market in state B, even without facing a run. Finally, a borrower that generates a default credit history D must be an L-type borrower. The deposit is risky in equilibrium, with deposit rate $r^I_S(D) = r_f/\pi$. The expected profit of financing such a loan is

$$\Pi_S(D) = \Pr(G)[R^*_S(D) - r^I_S(D)] = \Pr(G)R^*_S(D) - r_f.$$

Inserting the expressions of $\Pi^H_S(\overline{D})$, $\Pi^L_S(\overline{D})$ and $\Pi^L_S(D)$ into equation (25) and, after rearranging, we get

$$V_S = [\alpha + (1-\alpha)\delta\pi]R^*_S(\overline{D}) + (1-\alpha)(1-\delta)\pi R^*_S(D) - r_f.$$

The difference between the expected profits in the two regimes can be rewritten as follows

$$V_S - V_N = [\alpha + (1-\alpha)\delta\pi](R^*_S(\overline{D}) - R^*_N) + (1-\alpha)(1-\delta)\pi(R^*_S(D) - R^*_N) + \alpha(1-\pi)\rho R^*_N.$$

We now evaluate the difference $V_S - V_N$ in each region Ψ_j with $j = \{0, 1, 2, 3\}$. We indicate with φ_j the set of parameters that satisfy $V_S - V_N > 0$ in Case j.

Consider Case 0. We have $R_S^*(\overline{D}) = R_N^* = R_S^*(D) = R$ therefore

$$V_S - V_N = \alpha(1-\pi)\rho R > 0.$$

Therefore, $V_S > V_N$ always holds in region Ψ_0, so that $\varphi_0 = \Psi_0$.

Consider Case 1. We have $R_S^*(\overline{D}) = \frac{\alpha+(1-\alpha)\delta}{\alpha+(1-\alpha)\delta\pi}(c+r_f)$ and $R_N^* = R_S^*(D) = R$. Then

$$V_S - V_N = [\alpha + (1-\alpha)\delta]\,(c+r_f) - [(1-\alpha)\delta\pi + \alpha - \alpha(1-\pi)\rho]\,R.$$

Notice that $(1-\alpha)\delta\pi + \alpha - \alpha(1-\pi)\rho > 0$, so that $V_S - V_N > 0$ if and only if

$$R < \frac{\alpha + (1-\alpha)\delta}{\alpha - \alpha(1-\pi)\rho + (1-\alpha)\delta\pi}(c+r_f) \equiv R_1.$$

Notice that, given the definition of Ψ_1, $R_1 > R_S^E(\overline{D})$ always holds so that φ_1 is non-empty for $\forall\rho \in (0,1)$. On the other hand, φ_1 may not coincide with Ψ_1. Therefore, we have

$$\varphi_1 = \Psi_1 \bigcap \{R | R < R_1\} \subseteq \Psi_1.$$

Recall that the upper bound for R that defines Case 1 is given by R_N^E. If $R_1 > R_N^E$, the condition $V_S > V_N$ is satisfied in the entire region Ψ_1. This is true if and only if

$$R_1 = \frac{\alpha + (1-\alpha)\delta}{\alpha - \alpha(1-\pi)\rho + (1-\alpha)\delta\pi}(c+r_f) > \frac{1}{\alpha + (1-\alpha)\pi}(c+r_f) = R_N^E,$$

which implies

$$\rho > (1-\alpha)(1-\delta) \equiv \overline{\rho}.$$

Therefore, whenever the last inequality holds, φ_1 coincides with Ψ_1. Otherwise, we have $\varphi_1 \subset \Psi_1$. Notice that, given the definition of Ψ_1, φ_1 is always non-empty for $\forall\rho \in (0,1)$.

Consider Case 2. We have $R_S^*(\overline{D}) = \frac{\alpha+(1-\alpha)\delta}{\alpha+(1-\alpha)\delta\pi}(c+r_f)$, $R_N^* = \frac{1}{\alpha+(1-\alpha)\pi}(c+r_f)$ and $R_S^*(D) = R$, therefore

$$V_S - V_N = \left[\alpha + (1-\alpha)\delta - 1 + \frac{\alpha(1-\pi)\rho}{\alpha + (1-\alpha)\pi}\right](c+r_f) + (1-\alpha)(1-\delta)\pi R.$$

The condition $V_S - V_N > 0$ holds if and only if

$$R > \left[1 - \frac{1-\pi}{(1-\delta)}\frac{\alpha\rho}{(1-\alpha)[\alpha + (1-\alpha)\pi]}\right]\frac{(c+r_f)}{\pi} \equiv R_2.$$

Notice that, given the definition of Ψ_2, $R_2 < R_S^E(D)$ always holds so that φ_2 is non-empty for $\forall \rho \in (0,1)$. On the other hand, φ_2 may not coincide with Ψ_2. Therefore, we have

$$\varphi_2 = \Psi_2 \bigcap \{R|R > R_2\} \subseteq \Psi_2.$$

The lower bound for R that defines Case 2 is given again by R_N^E. If $R_2 < R_N^E$, $V_S > V_N$ is satisfied in the entire region Ψ_2. That is, if

$$R_2 = \left[1 - \frac{1-\pi}{(1-\delta)} \frac{\alpha\rho}{(1-\alpha)[\alpha + (1-\alpha)\pi]}\right] \frac{(c+r_f)}{\pi} < \frac{c+r_f}{\alpha + (1-\alpha)\pi} = R_N^E$$

which again implies

$$\rho > (1-\alpha)(1-\delta) = \overline{\rho},$$

we have $\varphi_2 = \Psi_2$. Otherwise, we have $\varphi_2 \subset \Psi_2$.

Consider Case 3. We have $R_S^*(\overline{D}) = \frac{\alpha+(1-\alpha)\delta}{\alpha+(1-\alpha)\delta\pi}(c + r_f)$, $R_N^* = \frac{1}{\alpha+(1-\alpha)\pi}(c + r_f)$ and $R_S^*(D) = \frac{1}{\pi}(c + r_f)$. It is straightforward to verify that $V_S = c$ and $V_N = c - \alpha(1 - \pi)\rho\frac{c+r_f}{\alpha+(1-\alpha)\pi}$. Therefore difference in expected profits is strictly positive

$$V_S - V_N = \alpha(1-\pi)\rho\frac{c+r_f}{\alpha + (1-\alpha)\pi} > 0,$$

so that $V_S > V_N$ is satisfied in the entire region Ψ_3. Therefore $\varphi_3 = \Psi_3$.

.8 Proof of Proposition 3

Suppose that the entrant bank, depositors and asset buyers all believe that the incumbent bank truthfully reveals the credit history of the borrower. Given that the borrower has previously defaulted, we check if the incumbent bank can profitably deviates by announcing a false credit history $\overline{D}$. We focus on the set of parameters $\varphi_j, j = \{0, 1, 2, 3\}$, for which the incumbent bank finds it profitable to share credit history ex ante. Recall that the incumbent bank does not fail in state B by misreporting the credit history since it can sell the loan at price $P_S^B(\overline{D}) > r_f$, so that in region $\varphi_j, j = \{0, 1, 2, 3\}$, we have $r_S(D) = r_f/\pi$ and $r_S(\overline{D}) = r_f$.

We first compute the incumbent bank's expected profit at $t = 1$ when truthfully reporting a loan with credit history D. That is,

$$\Pi_S(D|D) = \pi\left[R_S^*(D) - r_S(D)\right] = \pi R_S^*(D) - r_f. \tag{26}$$

The expected profit of reporting the false $\overline{D}$-history can be written as

$$\Pi_S(\overline{D}|D) = \pi R_S^*(\overline{D}) + (1-\pi)P_S^B(\overline{D}) - r_S(\overline{D}),$$

and given the expression for $P_S^B(\overline{D})$ we have

$$\Pi_S(\overline{D}|D) = \pi R_S^*(\overline{D}) + \frac{(1-\pi)\cdot\alpha\rho}{(1-\alpha)\delta+\alpha\rho} R_S^*(\overline{D}) - r_f. \tag{27}$$

Consider Case 0, where $R_S^*(D) = R_S^*(\overline{D}) = R$. We have

$$\Pi_S(D|D) = \pi R - r_f \tag{28}$$

and

$$\Pi_S(\overline{D}|D) = \pi R + \frac{(1-\pi)\cdot\alpha\rho}{\alpha\rho+(1-\alpha)\delta} R - r_f, \tag{29}$$

so that the incumbent bank finds it profitable to misreport the borrower's credit history. In this case, outsiders' belief cannot be rationalized and truthful information sharing cannot be sustained as a PBE in region φ_0.

Consider Case 1, where $R_S^*(D) = R$ and $R_S^*(\overline{D}) = \frac{\alpha+(1-\alpha)\delta}{\alpha+(1-\alpha)\delta\pi}(c+r_f)$. Reporting the true default history gives the same expected profit as in equation (28). The expected profit from deviation becomes

$$\Pi_S(\overline{D}|D) = \frac{\alpha\rho+(1-\alpha)\delta\pi}{\alpha\rho+(1-\alpha)\delta}\frac{\alpha+(1-\alpha)\delta}{\alpha+(1-\alpha)\delta\pi}(c+r_f) - r_f. \tag{30}$$

Then the ex-post incentive compatibility constraint to tell the truth is

$$\Pi_S(D|D) - \Pi_S(\overline{D}|D) = \pi R - \frac{\alpha\rho+(1-\alpha)\delta\pi}{\alpha\rho+(1-\alpha)\delta}\frac{\alpha+(1-\alpha)\delta}{\alpha+(1-\alpha)\delta\pi}(c+r_f) > 0,$$

which can be re-arranged as

$$R > \frac{1}{\pi}\left[\frac{\alpha\rho+(1-\alpha)\delta\pi}{\alpha\rho+(1-\alpha)\delta}\frac{\alpha+(1-\alpha)\delta}{\alpha+(1-\alpha)\delta\pi}\right](c+r_f) \equiv R_T. \tag{31}$$

Ex-ante information sharing is chosen in Case 1 when (recall the proof of Proposition 2)

$$R < \frac{\alpha+(1-\alpha)\delta}{\alpha-\alpha(1-\alpha)\rho+(1-\alpha)\delta\pi}(c+r_f) \equiv R_1.$$

It can be calculated that

$$\frac{1}{R_1} - \frac{1}{R_T} = \frac{1}{\alpha+(1-\alpha)\delta}\frac{\alpha^2(1-\rho)\rho(1-\pi)}{\alpha\rho+(1-\alpha)\delta\pi}\frac{1}{(c+r_f)} > 0$$

Consequently, $R_1 < R_T$ and there exists no R such that the incumbent bank will ex-ante participate in information sharing scheme and ex-post report the true default credit history. The belief of outsiders cannot be rationalized and truthful information sharing cannot be sustained as a PBE in region φ_1.

Consider Case 2. Since $R_S^*(D)$ and $R_S^*(\overline{D})$ remain the same as in Case 1, truthful reporting leads to the same expected profit as (26), and deviation leads to the same expected profit as in (30). Therefore, the incentive compatibility constraint for truth-telling is the same as in (31). Information sharing is ex-ante chosen in Case 2 when (recall proof of Proposition 2)

$$R > \frac{1}{\pi}\left[1 - \frac{1-\pi}{1-\delta}\frac{\alpha\rho}{(1-\alpha)[\alpha+(1-\alpha)\pi]}\right](c + r_f) \equiv R_2. \tag{32}$$

Since $R_2 < R_S^E(D)$ and $R_T < R_S^E(D)$ (more details in the proof for Case 3 below), there always exist a set of parameters (just below $R_S^E(D)$) such that the incumbent bank truthfully share the borrower's credit history in φ_2.

Furthermore, we establish a set of parameters in which, whenever it is ex-ante optimal for the incumbent bank to share information, it is also ex-post incentive compatible to disclose the true credit history, i.e. $R_2 > R_T$. This implies the following restriction

$$1 - \frac{1-\pi}{1-\delta}\frac{\alpha\rho}{(1-\alpha)[\alpha+(1-\alpha)\pi]} > \frac{\alpha\rho+(1-\alpha)\delta\pi}{\alpha\rho+(1-\alpha)\delta}\frac{\alpha+(1-\alpha)\delta}{\alpha+(1-\alpha)\delta\pi}. \tag{33}$$

Let us define the function

$$F(\rho) = 1 - \frac{1-\pi}{1-\delta}\frac{\alpha\rho}{(1-\alpha)[\alpha+(1-\alpha)\pi]} - \frac{\alpha\rho+(1-\alpha)\delta\pi}{\alpha\rho+(1-\alpha)\delta}\frac{\alpha+(1-\alpha)\delta}{\alpha+(1-\alpha)\delta\pi}$$

such that $R_2(\rho) = R_T(\rho)$ when $F(\rho) = 0$. It can be verified that

$$F'(\rho) = -\frac{1-\pi}{1-\delta}\frac{\alpha}{(1-\alpha)[\alpha+(1-\alpha)\pi]} - \frac{\alpha(1-\alpha)\delta(1-\pi)}{[\alpha\rho+(1-\alpha)\delta]^2}\frac{\alpha+(1-\alpha)\delta}{\alpha+(1-\alpha)\delta\pi} < 0.$$

Moreover, by taking the limits we have

$$\lim_{\rho\to 0} F(\rho) = 1 - \frac{\alpha\pi+(1-\alpha)\delta\pi}{\alpha+(1-\alpha)\delta\pi} > 0$$
$$\lim_{\rho\to 1} F(\rho) = -\frac{1-\pi}{1-\delta}\frac{\alpha}{(1-\alpha)[\alpha+(1-\alpha)\pi]} < 0.$$

Thus, there exists a unique $\underline{\rho} \in (0,1)$ such that $F(\underline{\rho}) = 0$ and inequality (33) holds for $\forall\rho \in (0, \underline{\rho})$. In this case, truth-telling can be sustained as a PBE in the whole region of φ_2. In fact, it can also be shown that $F(\overline{\rho}) < 0$ such that $\underline{\rho} < \overline{\rho}$.

Consider Case 3, where $R_S^*(D) = (c + r_f)/\pi$. Reporting the true default history leads to an expected profit

$$\Pi_S(D|D) = c.$$

The expected profit of misreporting the credit history is the same as in (30), and since

$$\frac{\alpha\rho + (1-\alpha)\delta\pi}{\alpha\rho + (1-\alpha)\delta} \frac{\alpha + (1-\alpha)\delta}{\alpha + (1-\alpha)\delta\pi} < 1$$

we have $\Pi_S(\overline{D}|D) < c = \Pi_S(D|D)$. Then truthful information sharing can be sustained as a PBE in the region φ_3.

.9 Proof of Corollary 1

We establish a unique $\underline{\rho}' \in (0, \underline{\rho})$ that makes $R_T(\underline{\rho}') = R_N^E$. Note that R_T increases in ρ,

$$\lim_{\rho \to 0} R_T(\rho) = \frac{\alpha + (1-\alpha)\delta}{\alpha + (1-\alpha)\delta\pi}(c + r_f) < R_N^E, \quad \text{and} \quad \lim_{\rho \to 1} R_T(\rho) = R_S^E(D) > R_N^E.$$

Therefore, we know $R_T(\underline{\rho}') = R_N^E$ defines a unique $\underline{\rho}'$. Further recall that, by definition, $\overline{\rho}$ satisfies $R_2(\overline{\rho}) = R_N^E$, and $\underline{\rho}$ satisfies $R_2(\underline{\rho}) = R_T(\underline{\rho})$. Given that R_2 decreases in ρ and $\underline{\rho} < \overline{\rho}$, we have

$$R_T(\underline{\rho}) = R_2(\underline{\rho}) > R_2(\overline{\rho}) = R_N^E = R_T(\underline{\rho}').$$

Since R_T increases in ρ, we have $\underline{\rho}' < \underline{\rho}$. The rest of the corollary follows.

Figure 2: Equilibrium loan rates: interior and corner solutions

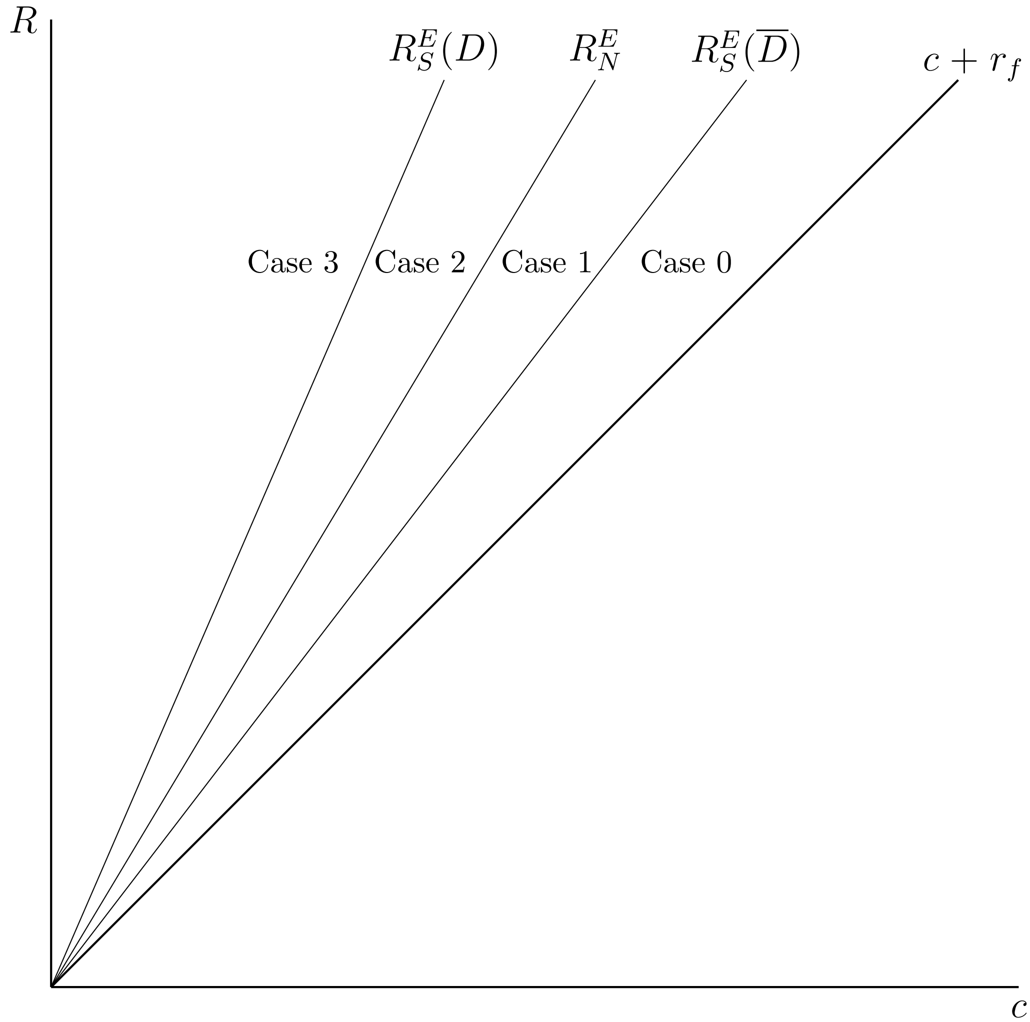
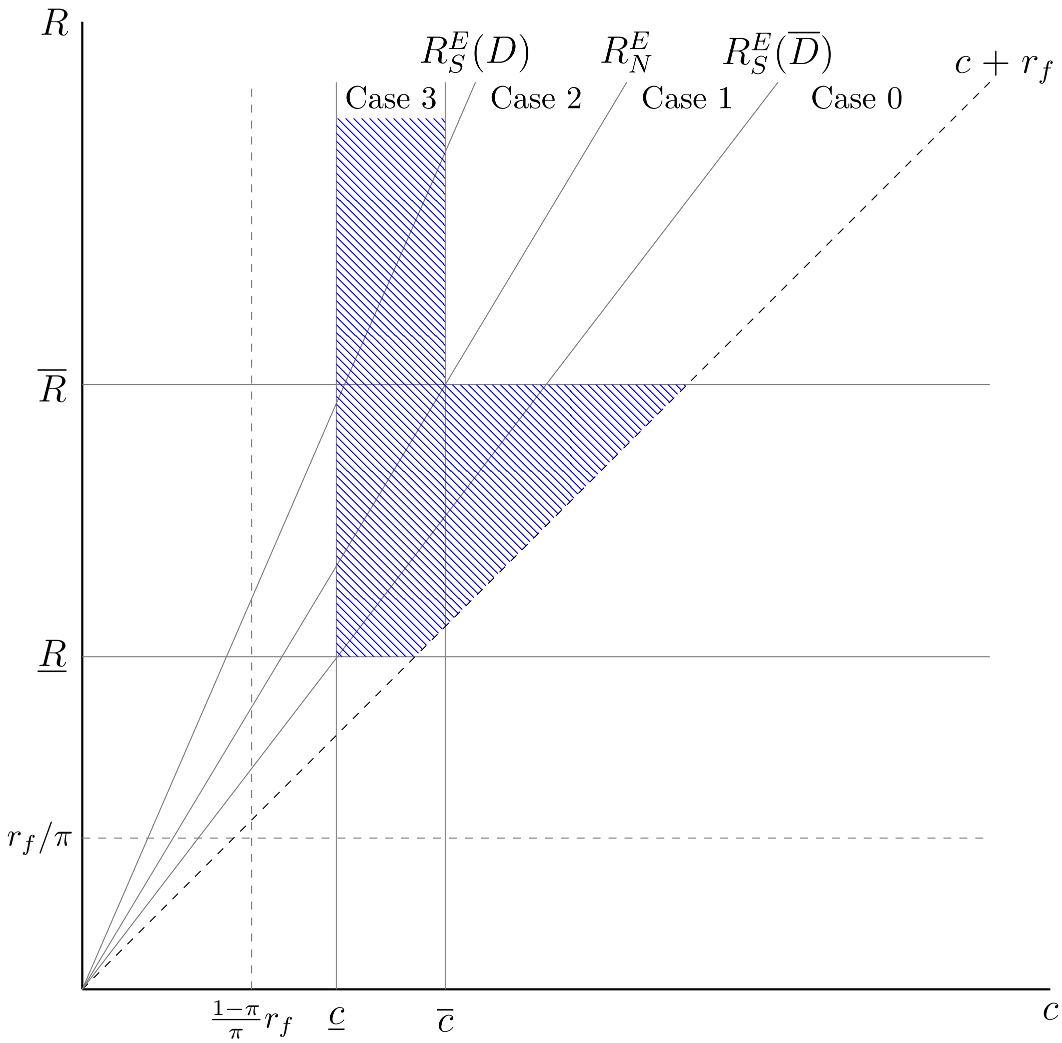
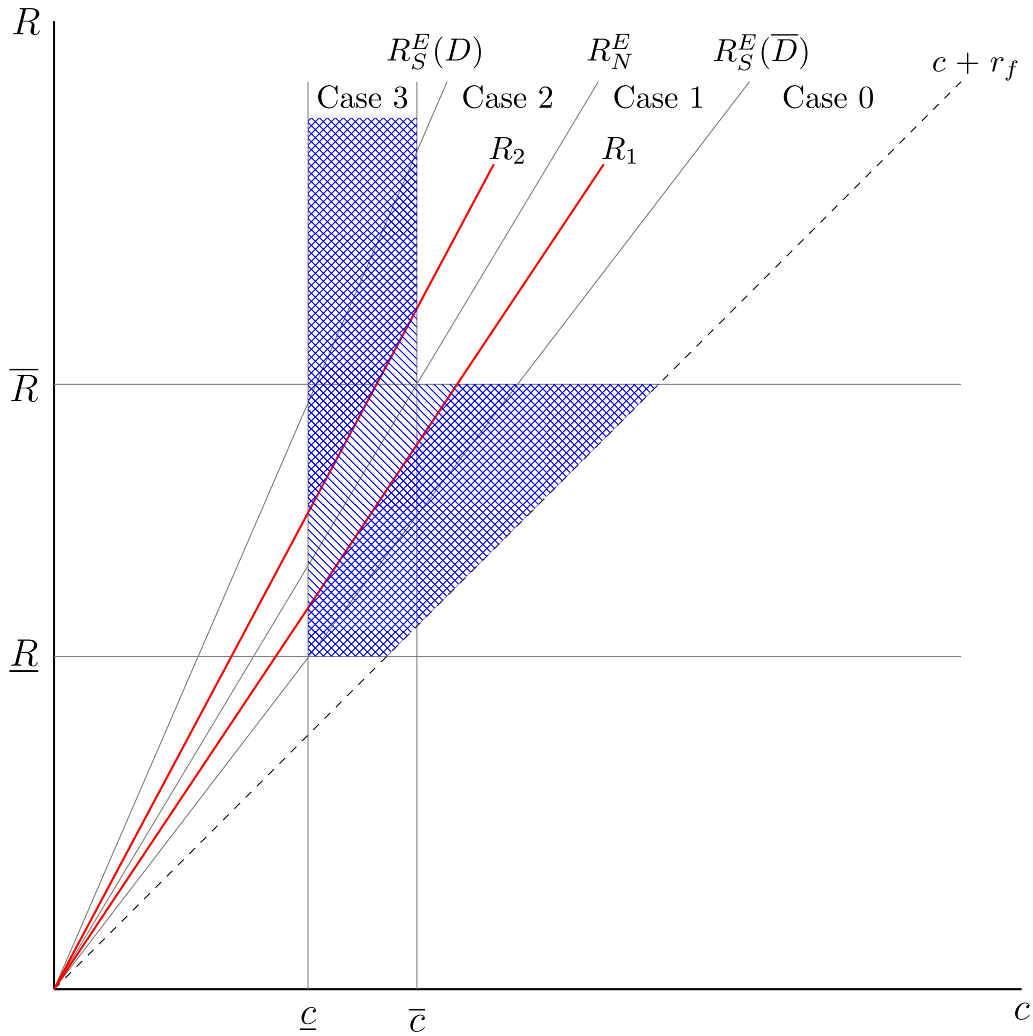


Figure 3: Regions where information sharing can save the incumbent bank from illiquidity



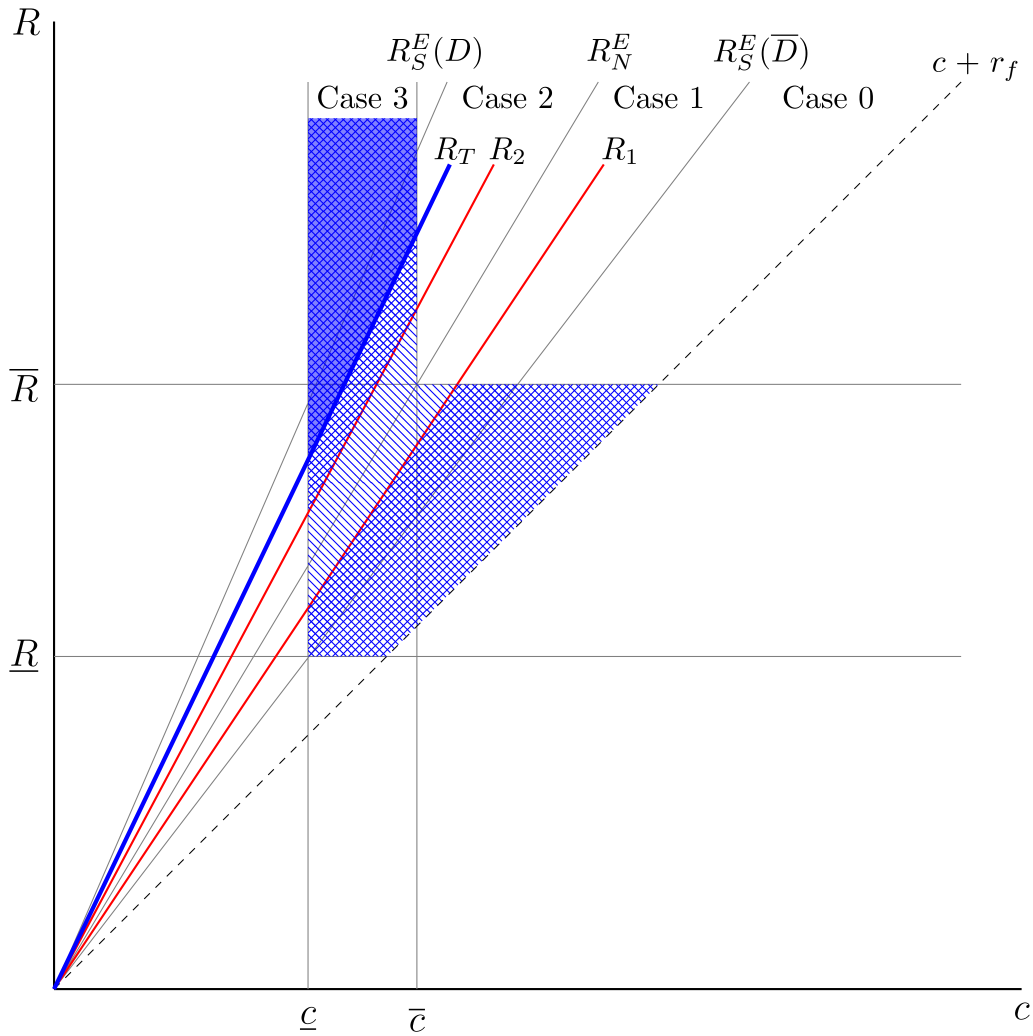
The region where information sharing can save the incumbent bank from illiquidity is indicated by the single-shaded area. The definitions of $\underline{c}$, $\bar{c}$, $\underline{R}$, and $\overline{R}$ can be found in Appendix A.5. The dashed lines indicate the parametric assumption (1)-(3). In particular, condition (1) is indicated by the region above the dashed line $R = c + r_f$; condition (2) is indicated by the region to the right of the dashed line $c = \frac{1-\pi}{\pi} r_f$; and condition (3) is indicated by the region above the dashed line $R = r_f/\pi$. When $\pi > \hat{\pi}$, $r_f/\pi < \underline{R}$ and $\frac{1-\pi}{\pi} r_f < \underline{c}$.

Figure 4: Regions where information sharing leads to a greater value for the incumbent bank



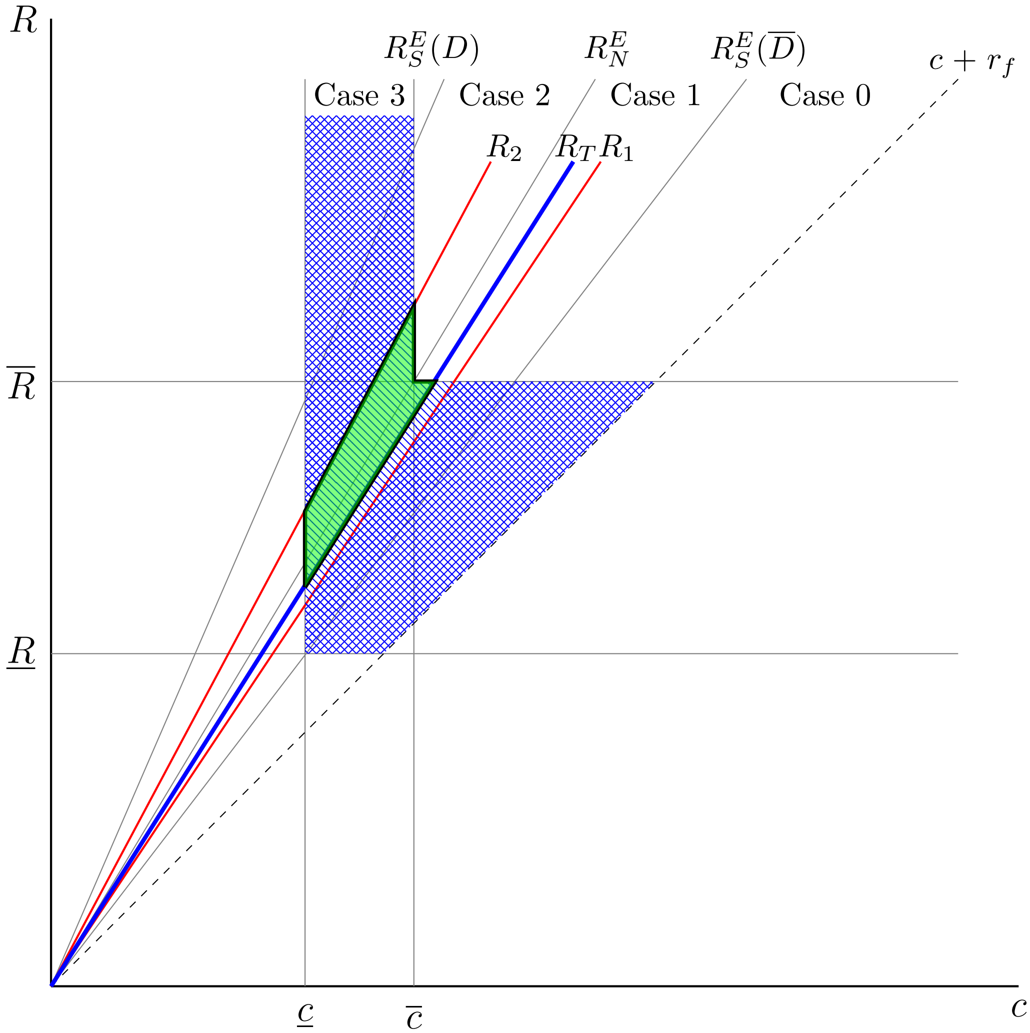
The region where information sharing leads to a greater value for the incumbent bank is indicated by the double-shaded area. Red line R_1 in Case 1 depicts the cutoff value *below* which information sharing increases the value of the incumbent bank, whereas red line R_2 in Case 2 depicts the cutoff value *above* which information sharing increases the value of the incumbent bank. We show a situation where $\rho < \overline{\rho}$, so that voluntary information sharing arises in a subset of Ψ_1 and Ψ_2. Graphically, the double-shaded area is smaller than the single-shaded area. In Case 0 and 3, voluntary information sharing arises in the entire region Ψ_0 and Ψ_3, so that the double-shaded area coincides with the single-shaded area. The definitions of R_1 and R_2 can be found in Appendix A.7.

Figure 5: Regions where truthful information sharing can be sustained in a perfect Bayesian equilibrium



Blue line R_T depicts the cutoff value *above* which truthful information sharing can be sustained in a perfect Bayesian equilibrium. The definitions of R_T can be found in Appendix A.8. The region where truth-telling can be sustained is indicated by the dark-blue area. We show a situation where $\underline{\rho} < \rho < \overline{\rho}$, so that truth-telling can be sustained in a subset of φ_2 in Case 2. In Case 0 and Case 1 there is no dark-blue area depicted since truth-telling is not sustainable under these two cases. The dark-blue area is then smaller than the double-shaded area. In Case 3 truth-telling is always sustained in the entire region φ_3 therefore the dark-blue area coincides with the double-shaded area.

Figure 6: Regions where a public registry can improve allocation



The region where a public registry can improve efficiency, subject to the truth-telling constraint, is indicated by the green area. The existence of the region is guaranteed when $\rho < \underline{\rho} < \overline{\rho}$. We show a situation where R_T is to the right of R_N^E, which is guaranteed for ρ that is sufficiently small. A public registry needs to be imposed, because the green region is beneath R_2 and the incumbent bank finds it unprofitable to share the borrower's credit history. The public registry can be sustained because the green region is above R_T, so that the bank has incentive to truthfully disclose the borrower's credit history, once sharing such information.